

Name: \_\_\_\_\_

MATH 2602 - Quiz 5  
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1. A certain lottery works as follows. Each lottery ticket has a 10-digit binary string (each digit is '0' or '1'). At the drawing, a 10-digit binary string is chosen at random, and a ticket wins if it agrees with the chosen string in at least 8 of the 10 digits. What is the probability that the first digit of a winning ticket agrees with the chosen string?

Let  $A$  be the outcome that a given ticket wins the lottery. Let  $B$  be the outcome that a given ticket matches the first digit of the chosen number. We are looking for  $P(B|A)$ .

$$P(B|A) = \frac{|A \cap B|}{|A|}.$$

Each of the 10 digits of the ticket can either agree or disagree with the chosen string, which gives  $2^{10}$  equally likely outcomes. For a ticket to win, either 0, 1 or 2 of its 10 digits can differ from the chosen string so

$$|A| = \binom{10}{0} + \binom{10}{1} + \binom{10}{2} = 56.$$

For a ticket to match the first digit and win, either 0, 1 or 2 of the 9 digits besides the first one can differ from the chosen string so

$$|A \cap B| = \binom{9}{0} + \binom{9}{1} + \binom{9}{2} = 46.$$

Then

$$P(B|A) = \frac{\binom{9}{0} + \binom{9}{1} + \binom{9}{2}}{\binom{10}{0} + \binom{10}{1} + \binom{10}{2}} = \frac{46}{56} = \frac{23}{28}.$$