

Name _____

Solution

Mathematics 2602

Quiz 9

Prof. Margalit

11 April 2012

Consider the linear system $Ax = b$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 16 \\ 3 & 9 & 40 \end{bmatrix}$ and $b = \begin{bmatrix} 14 \\ 60 \\ 141 \end{bmatrix}$.

Put the augmented matrix $(A | b)$ in row echelon form.

$$\begin{array}{c} [A|b] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 2 & 5 & 16 & 60 \\ 3 & 9 & 40 & 141 \end{array} \right] \xrightarrow{\begin{array}{l} r_2 \leftarrow (-2)r_1 + r_2 \\ r_3 \leftarrow (-3)r_1 + r_3 \end{array}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & 1 & 10 & 32 \\ 0 & 3 & 31 & 99 \end{array} \right] \\ \xrightarrow{r_3 \leftarrow (-3)r_2 + r_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & 1 & 10 & 32 \\ 0 & 0 & 1 & 3 \end{array} \right]. \end{array}$$

Find the reduced row echelon form for $(A | b)$.

$$\begin{array}{c} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & 1 & 10 & 32 \\ 0 & 0 & 1 & 3 \end{array} \right] \xrightarrow{\begin{array}{l} r_1 \leftarrow (-3)r_3 + r_1 \\ r_2 \leftarrow (-10)r_3 + r_2 \end{array}} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right] \\ \xrightarrow{r_1 \leftarrow (-2)r_2 + r_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]. \end{array}$$

Solve the system $Ax = b$ for x .

The system $Ax = b$ is equivalent to

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \text{ so } x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$