

Mathematics 2602

Quiz 1

Prof. Margalit

16 January 2013

1. Show that the following conditional statement is a tautology:

$$(\neg a \vee c) \wedge (a \vee b) \rightarrow (b \vee c).$$

You can either use a truth table or the basic rules of logical equivalence.

Truth table

a	b	c	$\neg a$	$\neg a \vee c$	$a \vee b$	$(\neg a \vee c) \wedge (a \vee b)$	$b \vee c$	$(\neg a \vee c) \wedge (a \vee b) \rightarrow (b \vee c)$
T	T	T	F	T	T	T	T	T
T	T	F	F	F	T	F	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	F	T

By manipulation:

$$(\neg a \vee c) \wedge (a \vee b) \rightarrow (b \vee c)$$

$$\Rightarrow (\neg a \vee c) \wedge (a \vee b) \vee (b \vee c) \quad \text{-if then equivalence}$$

$$\Rightarrow \overline{\neg a \vee c} \vee \overline{a \vee b} \vee (b \vee c) \quad \text{De Morgan's}$$

$$\Rightarrow (a \wedge \neg c) \vee (\neg a \wedge \neg b) \vee (b \vee c) \quad \text{De Morgan's}$$

$$\Rightarrow [c \vee (a \wedge \neg c)] \vee [b \vee (\neg a \wedge \neg b)] \quad \text{re-ordering}$$

$$\Rightarrow [(c \vee a) \wedge (c \vee \neg c)] \vee [(b \vee \neg a) \wedge (b \vee \neg b)] \quad \text{distributing}$$

$$\Rightarrow [(c \vee a) \wedge T] \vee [(b \vee \neg a) \wedge T] \quad \text{use } p \vee \neg p \equiv T$$

$$\Rightarrow (c \vee a) \vee (b \vee \neg a) \quad \text{use } p \wedge T \equiv p$$

$$\Rightarrow c \vee b \vee (a \vee \bar{a}) \quad \text{commutativity and associativity}$$

$$\Rightarrow c \vee b \vee T \equiv T$$

$$\text{use } p \vee \neg p \equiv T \text{ and } p \vee T \equiv T \quad \square$$