ANNOUNCEMENTS APR 1

- Cameras on
- HW due Thu 3:30
- OH Fri 2-3, Tue 11-12, appt
- Outline due Fri nite
- Makeup points

Today

Thompson's group.

Thompson's gp F

F = group of assoc. laws

or: How to get between all parenthesizations of an expression

x_0 : $a(bc) \to (ab)c$

x_1 : $a(b(cd)) \to a((bc)d)$

x_2 : $a(b(c(de))) \to a(b((cd)e))$

subscript $\leftrightarrow$ "depth"

Composition: do first, then the second.

Must allow for interpreting a, b, c as expressions themselves and allow expansions

$\ldots a \ldots \to \ldots (a_1 a_2) \ldots$

A, B, C expressions in a, b, c. $\longrightarrow$ $(A)(BC) = (AB)(C)$

example. $\underbrace{a}_{A}(\underbrace{b}_{B}\underbrace{(c(de))}_{C}) \xrightarrow{x_0} (ab)(c(de))$

A relation:

$$\begin{tikzcd}
a(b(c(de))) \arrow[r, "x_0"] \arrow[d, "x_2"'] & (ab)(c(de)) \arrow[d, "x_1"] \\
a(b((cd)e)) \arrow[r, "x_0"] & (ab)((cd)e)
\end{tikzcd}$$

So: $x_1 x_0 = x_0 x_2$ (right to left mult)

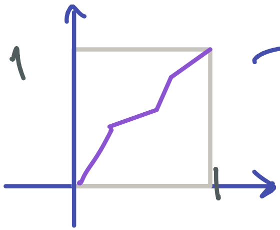
More generally:

$x_n x_i = x_i x_{n+1}$ $i < n$.

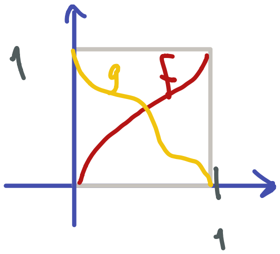
Q Is this really a group?

F via PL homeos

$F = \{$ orientation preserving, piecewise linear homeomorphisms of $[0,1]$ with dyadic break points with slopes powers of 2 $\}$ under fn composition.



Homeos A fn $f : [0,1] \to [0,1]$ is a homeo if it is contin with contin. inverse.



f, g are both homeos.

Orient. pres A homeo $f : [0,1] \to [0,1]$ is or. pres. if $f(0) = 0$.

Piecewise liner What you think. (finitely many line segments)

F via PL homeos

$F = \{$ orientation preserving,
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of $[0,1]$ with
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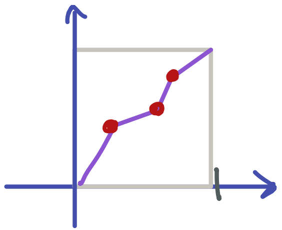
break points
with slopes powers of $2\}$

under fn composition.

Break points



Dyadic $m/2^k$ $k, m \in \mathbb{Z}$

Why is this a group?

think about composition

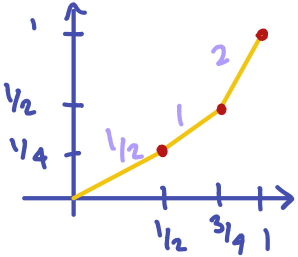
or pres ✓

homeo ✓

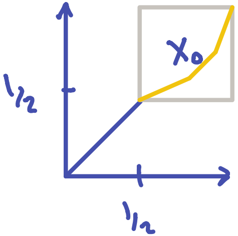
break pts dyadic: exercise.

slopes powers of 2: chain rule

Examples

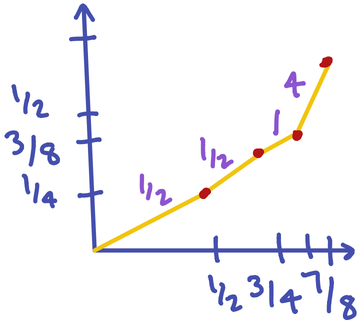


x_0



x_1

What is $x_1 x_0$?



Where is 15/16?

Break pts: Break pts of x_0

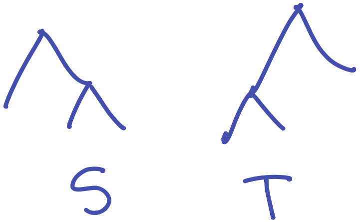
$\cup$ x_0^{-1}(Break pts of x_1)

$$\{1/2, 3/4\} \cup \underbrace{x_0^{-1}(\{1/2, 3/4, 7/8\})}_{\{3/4,\ 7/8,\ 15/16\}}$$

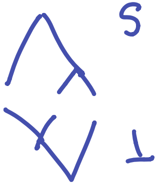
F via tree pairs

A tree pair is a pair of binary trees with same # of leaves

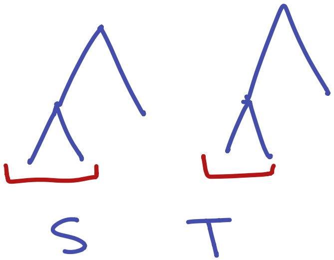
e.g.



or



Reduced if no canceling carets:



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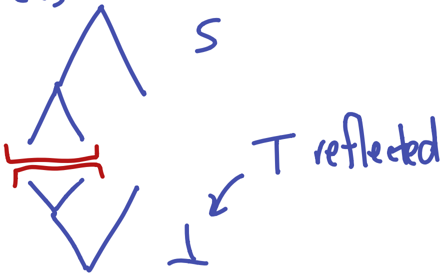

F = {reduced tree pairs}

Mutiplication:

$$(S_2, T_2)\cdot(S_1, T_1)$$

Is (S_2, T_1) if $T_2 = S_1$

If $T_2 \neq S_1$, add carets until they are equal.

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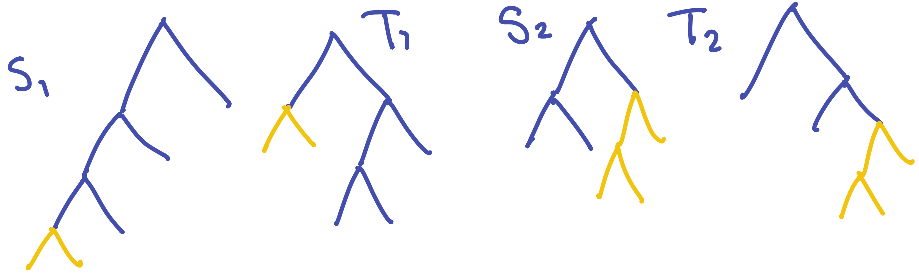
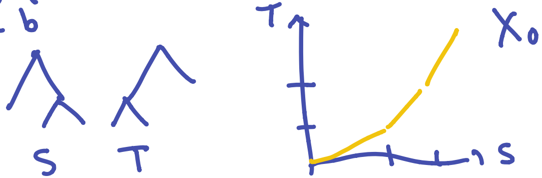


Carets $\longleftrightarrow$ parentheses.

$\longleftrightarrow$ dividing interval in half



Some Facts about F

① F is gen. by x_0, x_1

② F is finitely pres.

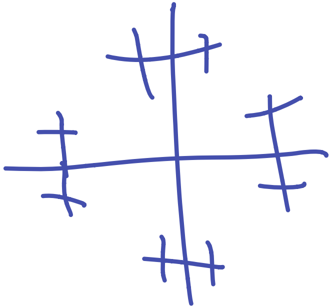
③ F contains ④ F

$\cancel{2}$ $\cancel{\alpha}$ ∞

Major open question

Q. Is F amenable?

A group is non-amenable if its Cayley graph admits a Ponzi scheme.



everyone passes \$1 to inward neighbor.

Yes $\Rightarrow$ F is a fin pres amen gp that is not elem. amenable.

No $\Rightarrow$ F is a fin pres non-amen gp with no free subgp.