Math 6441

See web page from my web page (Dan Margalit)

Please wear masks.

Grading: HW 60%.

Midterm 20%.

Final Project 20%.

Book: Hatcher.

Recordings: Math 6441 → Lectures → Files → Recordings

Office Hours: TBA.

Math 6441 : Alg Top

Jan 10

What is alg. top?

Space → Group

hard → easier

$X \rightleftarrows \pi_1(X)$ fundamental gp.

abelian:

$X \longrightarrow H_k(X)$ k-th homology group

$X \longrightarrow H^k(X)$ k-th cohomology gp.

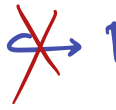
What kinds of questions does it answer?

① When are two spaces the same?

e.g. $\mathbb{R}^n \not\cong \mathbb{R}^m$ $m \neq n$.

or $\mathbb{R}^3 -$  $\not\cong \mathbb{R}^3 -$ 

② Embeddings

e.g. Klein bottle  $\mathbb{R}^3$

More general:

What is smallest N so a given manifold embeds in $\mathbb{R}^N$.

Unsolved for $\mathbb{RP}^n$.

③ Fixed pt theorems

Brouwer fixed pt thm:

every contin. $D^2 \to D^2$ has a fixed pt.

Borsuk-Ulam thm.

Any contin $S^2 \to \mathbb{R}^2$ has antipodal pts with some image.

④ Actions

Which finite gps act freely on S^n? Known in some cases

Note: $\mathbb{Z}/n \curvearrowright S^{2k-1}$ freely.

⑤ Sections

What is the largest k s.t. a given manifold admits a continuously varying k-plane field?

example: Can't-comb-a-monkey-theorem

⑥ Group theory

- Every subgp of a free gp is free.
- $[F_n, F_n]$ is not fin. gen.
- Braid groups are torsion free

⑦ Algebra

Fund thm of algebra

$\mathbb{R}^3$ is not a field.

⑧ Graph theory.

A convex polyhedron made of triangles is 3-colorable iff it has an even # of triangles at each vertex (Kontsevich proof).

Also: Robotics

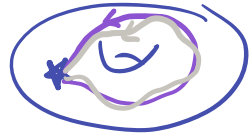
Networks

Data science

Exotic manifolds.

Outline/Overview

Ⅰ Fundamental Group



elements: loops at basept/$\sim$

operation: concat.

We'll see:

groups $\longleftrightarrow$ spaces

subgps $\longleftrightarrow$ covering spaces

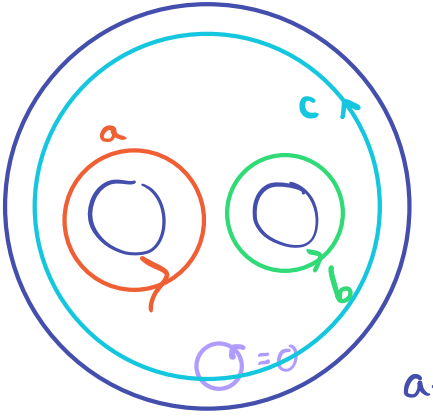
homoms $\longleftrightarrow$ maps.

II Homology

Idea:

$H_k(X)$ = abelian gp of k-dim holes in X

X = pair of pants 



$H_1(X) \cong \mathbb{Z}^2$

$a+b=c$.

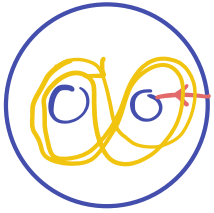
III Cohomology

$H^k(X)$ dual to $H_k(X)$

Consists of functions

$H_k(X) \to \mathbb{Z}$

$\rightsquigarrow$ ring, not just abel. gp.



Big goal: Poincaré Duality

For X = n-manifold

$$H^k(X) \cong H_{n-k}(X)$$

More precisely, the functions in $H^k(X)$ are: intersect with some elt of $H_k(X)$