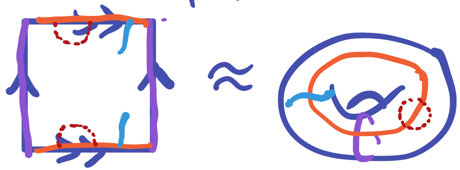
What do we mean by space?

Cell complexes a.k.a.

CW complexes

e.g. 

Basically: glue cells together

Quotient topology:

$U \subseteq X/\sim$ open iff

its preimage in X open

Jan 12

We build a CW complex inductively:

(i) Start with a discrete set of pts X^0

"0-cells" 0-skeleton

(ii) Inductively form n-skeleton X^n

from X^{n-1} by attaching n-cells

D^n_α via $\varphi_\alpha : \partial D^n_\alpha \to X^{n-1}$

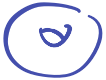
n-ball

index

In torus example:

$X^0 = \bullet$

$X^1 =$  

$X^2 =$ 

exercise: write φ_α

Can Either stop at a finite stage, or continue indefinitely.

Topology is the weak topology: a set is open iff its intersection with each cell is open.

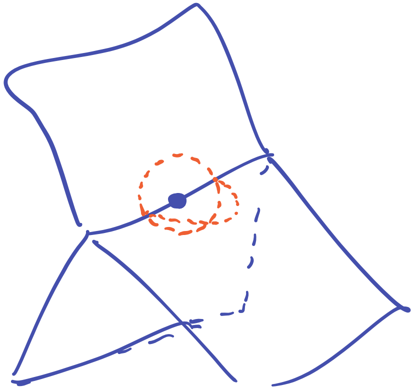
$$\dim X = \sup\{\text{dim of cells}\}$$

If $\dim X < \infty$

weak top $\equiv$ quotient top.

exercise. Think about continuous paths.

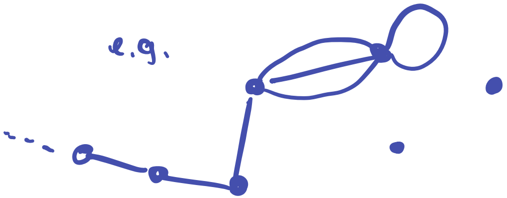
an open set



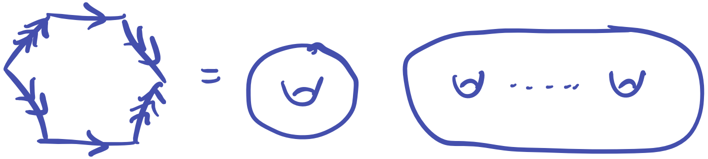
Examples

① 1-dim CW complexes are graphs

e.g.



② $(4g+2)$-gon with opp. sides identified



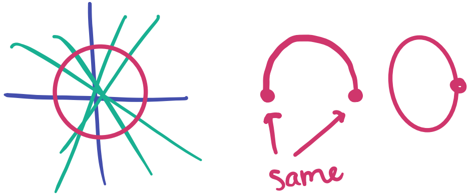
$g=1$

③ $S^n = e^0 \cup e^n$

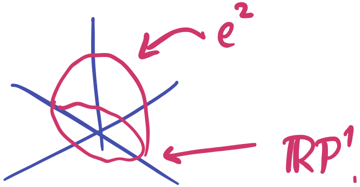
④ $\mathbb{RP}^n$ = space of lines thru 0 in $\mathbb{R}^{n+1}$

$= e^0 \cup e^1 \cup \cdots \cup e^n$.

$\mathbb{RP}^1$: $= S^1/\text{antipod.}$



$\mathbb{RP}^2$: $= S^2/\text{antipod}$



(5) $\mathbb{C}P^n$ = space of lines thru 0 in $\mathbb{C}^{n+1}$

$= e^0 \cup e^2 \cup e^4 \cup \cdots \cup e^{2n}$

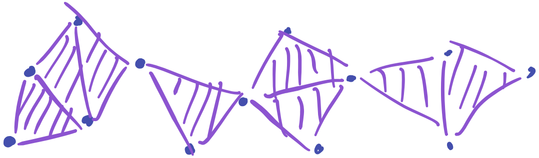
$\mathbb{C}P^1 = \hat{\mathbb{C}} \approx S^2$

Also: can have $n = \infty$ in last 2 examples!

(6) All smooth closed manifolds

(7) All top. closed manifolds of dim $\neq 4$ (Manolescu)

(8) Networks/Data (Čech complexes)



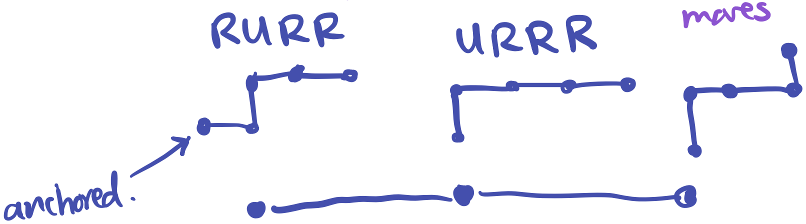
(9) Robot arms (Ghrist)

(or, seqs of U's & R's)

edges • swap RU, UR

• change last letter.

square • "commuting"/disjoint moves



exercise: draw the whole thing.