Groups Acting on Connected Complexes

Lemma. $G \circlearrowright X$ = connected graph transitive on vertices and ordered pairs of vertices

Say $v \bullet\!\!-\!\!\bullet w$ & $h \cdot w = v$

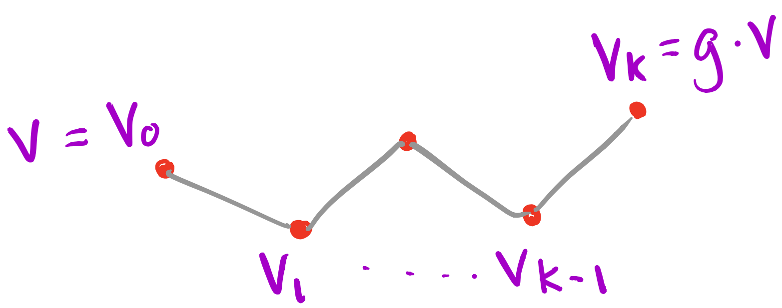
Then: $G = \langle \mathrm{Stab}_G(v), h \rangle$

Pf. Let $H = \langle \mathrm{Stab}_G(v), h \rangle$

$g \in G$.

Want to show $g \in H$.

Consider a path



Choose g_i s.t. $v_i = g_i \cdot v$

& $g_0 = \mathrm{id}$, $g_k = g$.

Inductive hyp: $g_i \in H$.

Base case automatic. Assume $g_i \in H$.

Consider $v_i = g_i \cdot v$ — $v_{i+1} = g_{i+1} \cdot v$

Apply g_i^{-1}: v — $g_i^{-1} g_{i+1} \cdot v$

Since G acts trans. on pairs of vertices...

Apply some r: v — $w = r g_i^{-1} g_{i+1} \cdot v$

Note: $r \in \mathrm{Stab}_G(v)$

$\rightsquigarrow \quad h r g_i^{-1} g_{i+1} \in \mathrm{Stab}_G(v) \subseteq H$

But $h, r, g_i^{-1} \in H \implies g_{i+1} \in H$. $\square$

Proof of Finite Generation

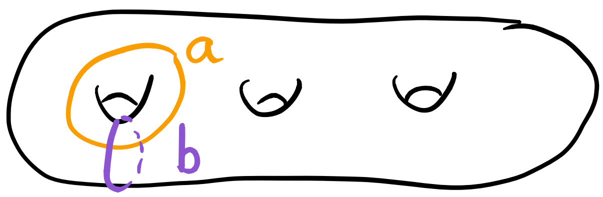
<u>Theorem</u>. For $g \geq 0$ $\mathrm{Mod}(S_g)$ is finitely gen. by Dehn twists.

<u>Proof</u>. Induct on g. Base cases $g=0,1$ ✓

Let $g \geq 2$.

$\mathrm{Mod}(S_g) \circlearrowright \hat{N}(S_g)$ satisfying Lemma.

Let



Check: $T_a T_b T_a(b) = a$

Lemma $\Rightarrow$ $\mathrm{Mod}(S_g) = \langle \mathrm{Stab}(a), T_a, T_b \rangle$

To show $\mathrm{Stab}(a)$ fin. gen. by Dehn twists.

$\mathrm{Stab}(a)/\langle T_a \rangle \cong \mathrm{Mod}(S_{g-1,2})$

cut along a.

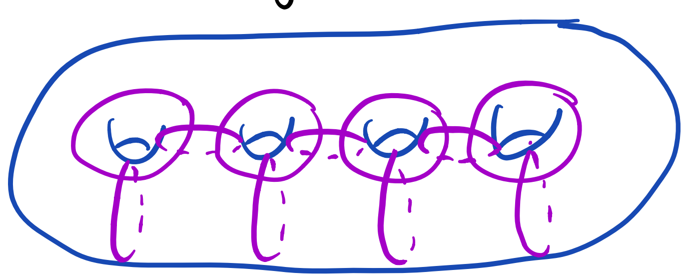
By induction $Mod(S_{g-1})$ fin. gen. by Dehn twists.

$\Longrightarrow$ $Mod(S_{g-1,1})$ fin. gen. by Dehn twists

(Birman exact seq + usual gen. set for $\pi_1(S_g)$)

$\Longrightarrow$ $Mod(S_{g-1,2})$ fin. gen by DTs $\square$

Same proof:
- Finite gen. by DTs about nonsep curves (since π_1 gen by nonsep simple loops)
- Lickorish generators



Just check that each step works!