Problems on Torelli Groups

Generation

- Find a conceptual proof $\mathcal{I}(S_3)$ is fin. gen.
- Is there a gen set for $\mathcal{I}(S_g)$ with $\binom{2g+1}{3}$ elts?
- Find an explicit gen set for $\mathcal{I}(S_2)$.
- Find an explicit finite gen set for $\mathcal{K}(S_g)$.
  Other terms of Johnson filtration, terms of LCS.
- Is $\mathcal{K}(S_3)$ finitely generated?
- Is $\mathcal{K}(S_g)$ generated by twist differences $T_c T_d^{-1}$?
  Other terms of Johnson filtration?
- Find explicit gen sets for terms of Johnson filtration.

Relations

- Is $\mathcal{I}(S_g)$ finitely presented?

  What about $\mathcal{K}(S_g)$, terms of Johnson filtration, etc.?

- Show that two elements of $\mathcal{I}(S_g)$ either commute or generate a free group.

  Or just do it for BP maps, or just for $\mathcal{K}(S_g)$.

Cohomology

- Compute $H_k(\mathcal{I}(S_g))$

  Is it f.g. for any $g \geq 3$ $2 \leq k < 2g-3$?

- What is cohomological dimension of terms of Johnson filtration?

- Show the largest free abelian subgp has rank $g-1$.

- Show $H_{2g-3}(\mathcal{K}(S_g))$ is ∞-gen.

Hyperelliptic Torelli

- Find a simpler proof that $\mathcal{SI}(S_g)$ is gen. by Dehn twists.

- Is $\mathcal{SI}(S_g)$ finitely generated? presented?

- Is $\mathcal{SI}(S_g)^{ab}$ finitely generated?

- Find natural gen sets for congruence subgroups $\mathrm{SMod}(S_g)[m]$. What are the abelianizations?

Stretch Factors

- Improve the gap between $\sqrt{2}$ & 62 for the smallest stretch factor in $\mathcal{I}(S_g)$.

- Is the smallest stretch factor in $\mathcal{I}(S_g)$ smaller than that in $\mathcal{K}(S_g)$? Further terms of Johnson filtration?

- Which algebraic degrees for stretch factors arise in $\mathcal{I}(S_g)$?

- What is the smallest stretch factor in $\mathrm{Mod}(S_g)[2]$?

EMBEDDINGS

- Show that any embedding $\mathcal{I}(S_g) \hookrightarrow \mathrm{Mod}(S_g)$ is standard.
- Show any non-abelian map $\mathcal{I}(S_g) \longrightarrow \mathrm{Mod}(S_k)$ is trivial if $g \neq k$.
- Similar for $\mathcal{K}(S_g)$, other terms of Johnson filtration.

MISCELLANEA

- Find a simple description of BCJ maps, using double covers.
- Find the image of the second Johnson homomorphism.
- Which subsets of $\mathcal{I}(S_g)$ give all $\mathbb{Z}HS^3$s?
- Determine if the even MMM classes vanish on $\mathcal{I}(S_g)$.