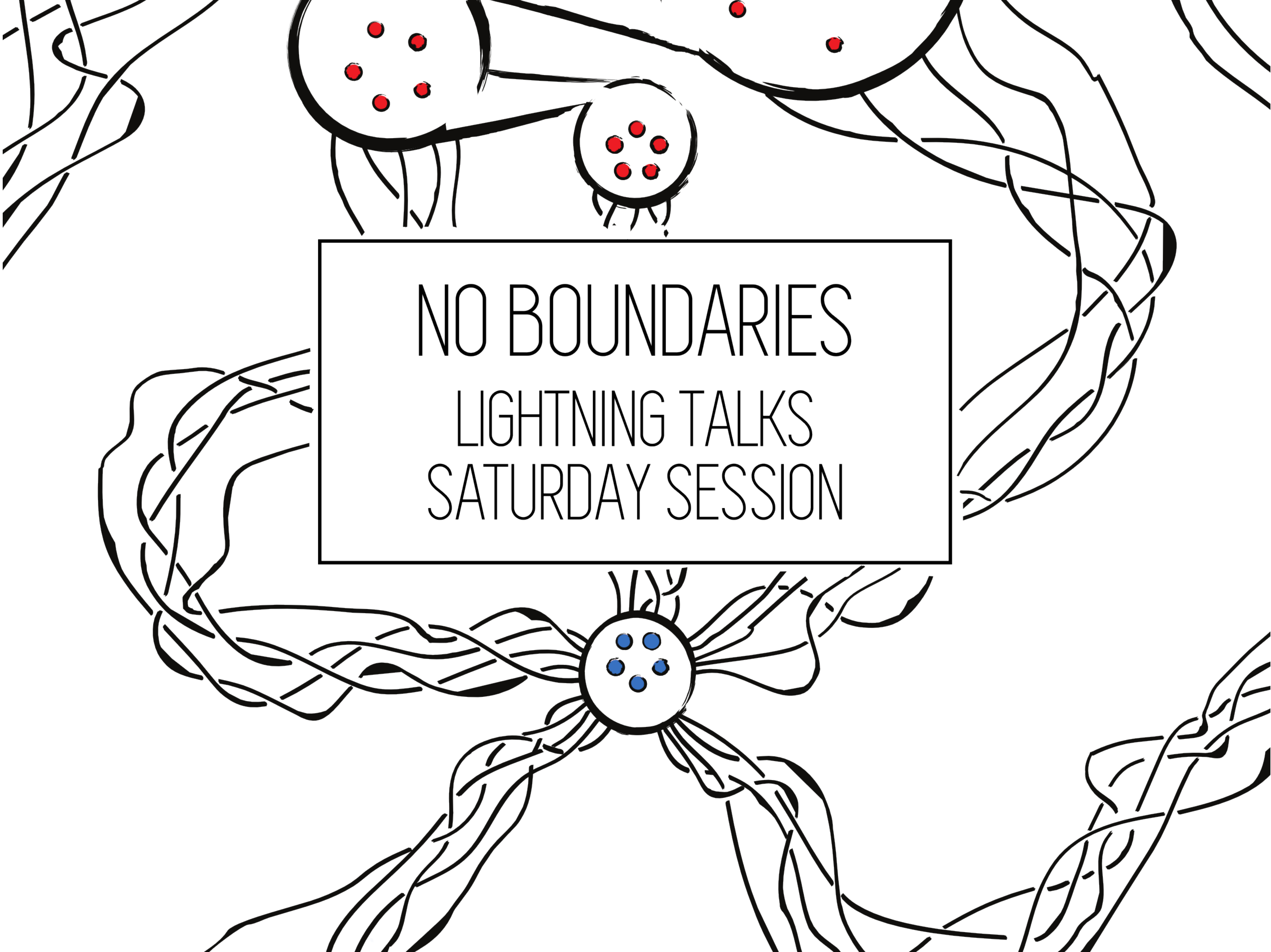
NO BOUNDARIES

LIGHTNING TALKS
SATURDAY SESSION



Twisted rabbits and Hubbard trees

Becca Winarski

University of Wisconsin-Milwaukee

joint with Jim Belk, Justin Lanier and Dan Margalit

The twisted rabbit problem

$$p(z) = z^2 + c$$

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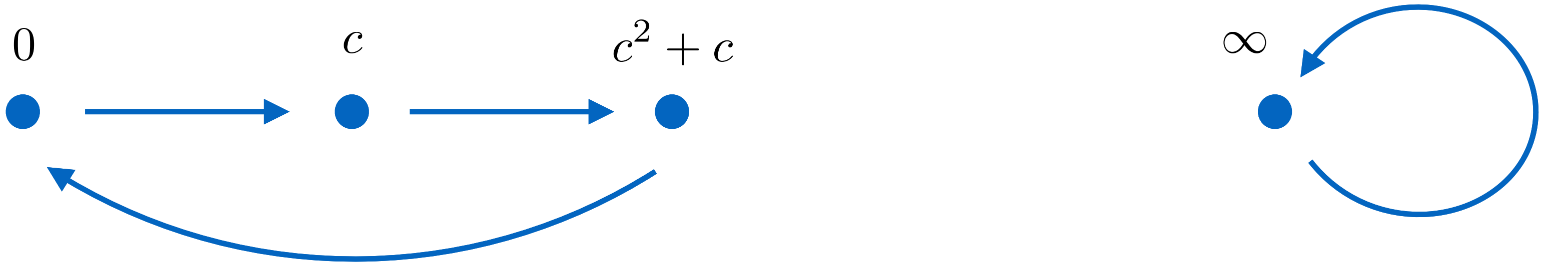
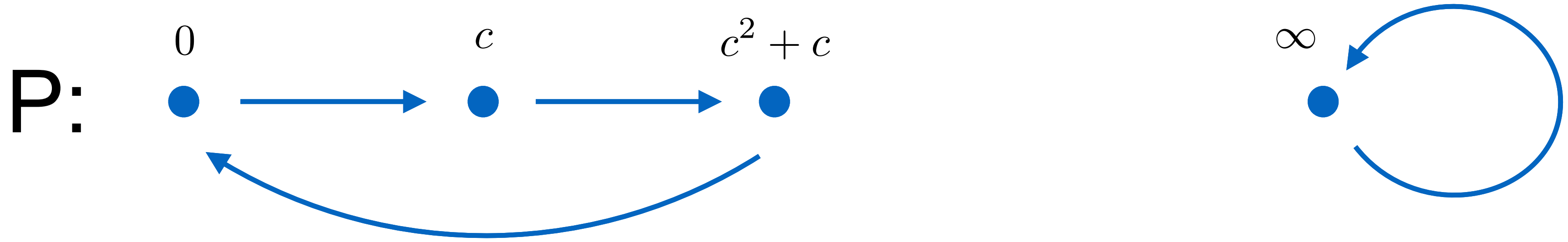
P:



The twisted rabbit problem

$$p(z) = z^2 + c$$

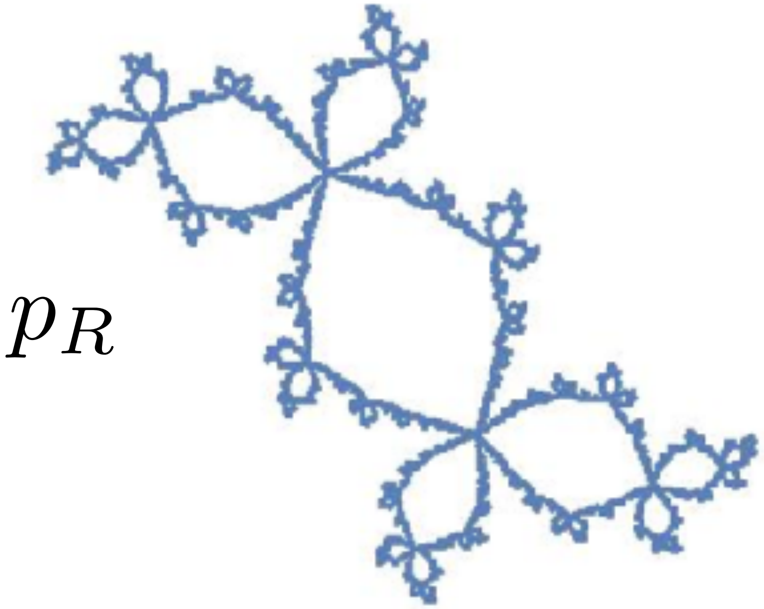


3 values of c → p_R, p_C, p_A

Julia sets

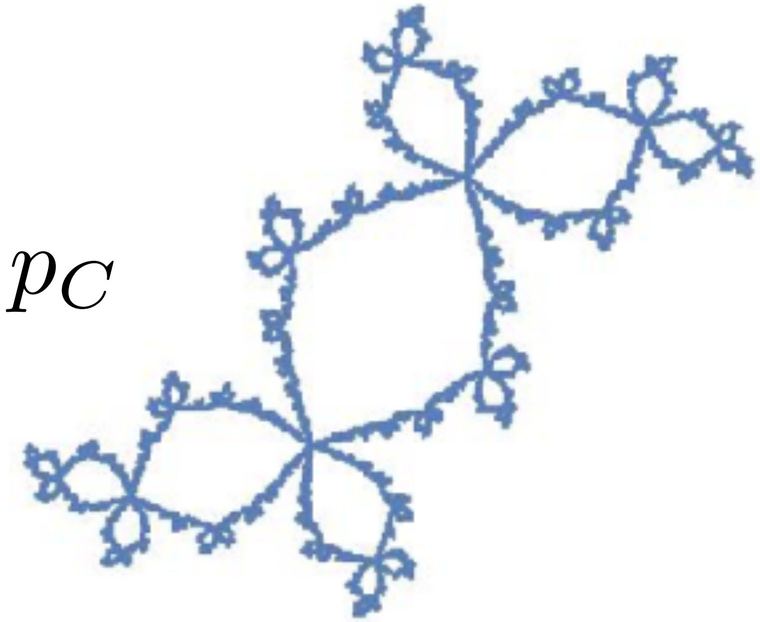
Rabbit
Im(c)>0

p_R



Corabbit
Im(c)<0

p_C

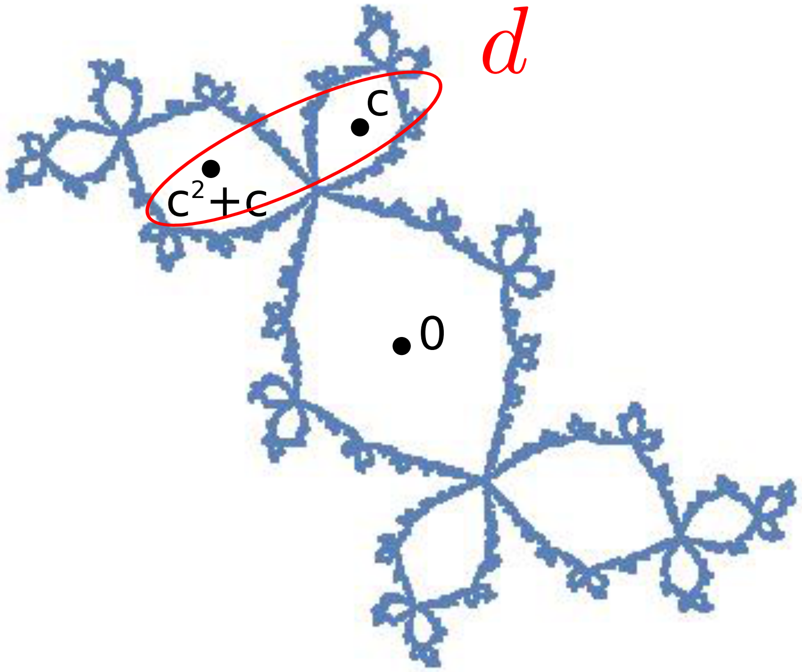


Airplane
Im(c)=0

p_A

Images courtesy of Bill Floyd https://www.math.vt.edu/netmaps/index.php

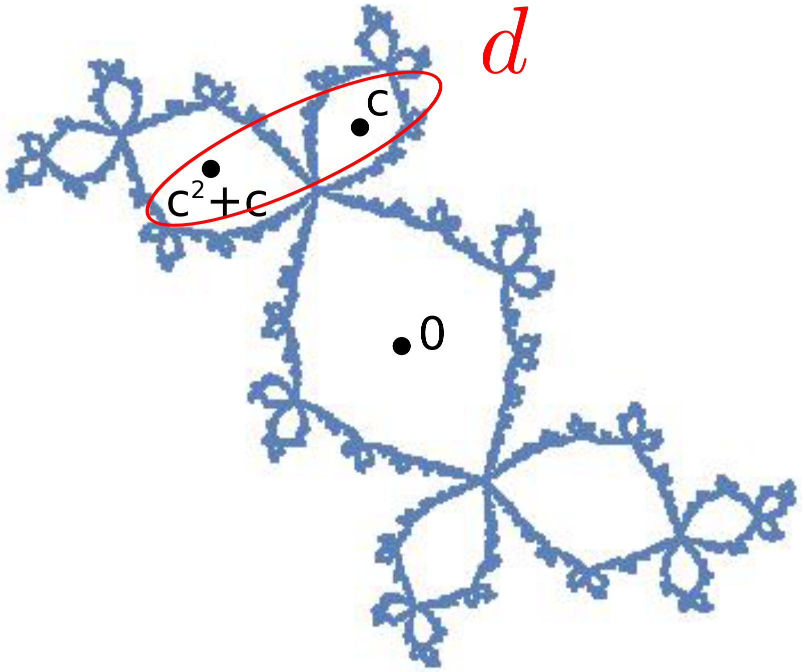
The twisted rabbit problem



p_R

The twisted rabbit problem

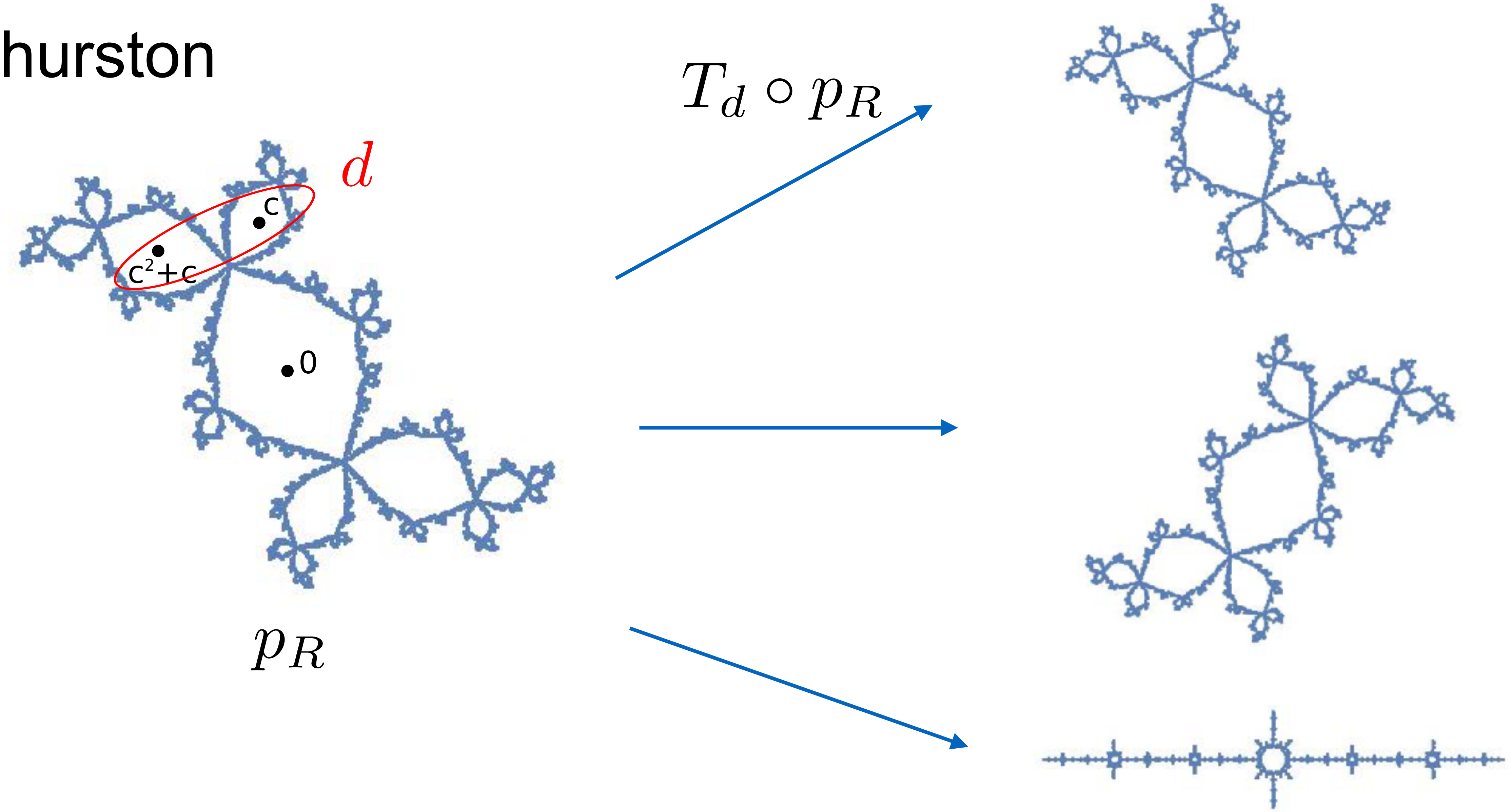
$T_d \circ p_R$



p_R

The twisted rabbit problem

Thurston



The twisted rabbit problem

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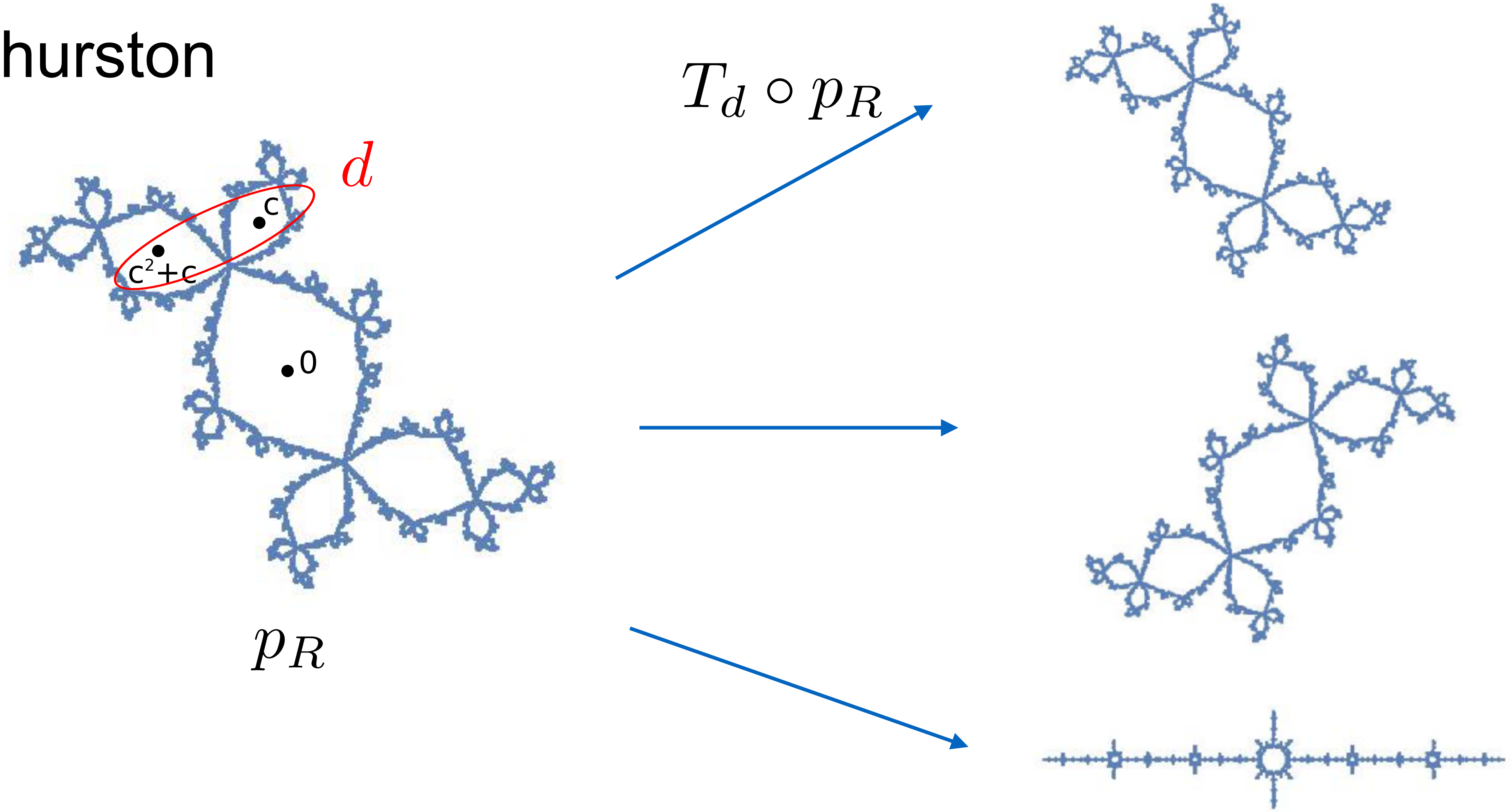

Twisted rabbit problem:

$f \in \mathrm{Mod}(\mathbb{C}, P)$ what is $f \circ p_R$?

Solving TRP

1. Topological description of p_R

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2. Distinguish p_R, p_C, p_A

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2. Distinguish p_R, p_C, p_A → Hubbard trees

3. Given f, what is $f \circ p_R$?

Solving TRP

1. Topological description of p_R → branched covers

2. Distinguish p_R, p_C, p_A → Hubbard trees

3. Given f, what is $f \circ p_R$?

→ following Bartholdi—Nekyrashevych

Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:

- edges are contained in Julia set
- leaves are in P

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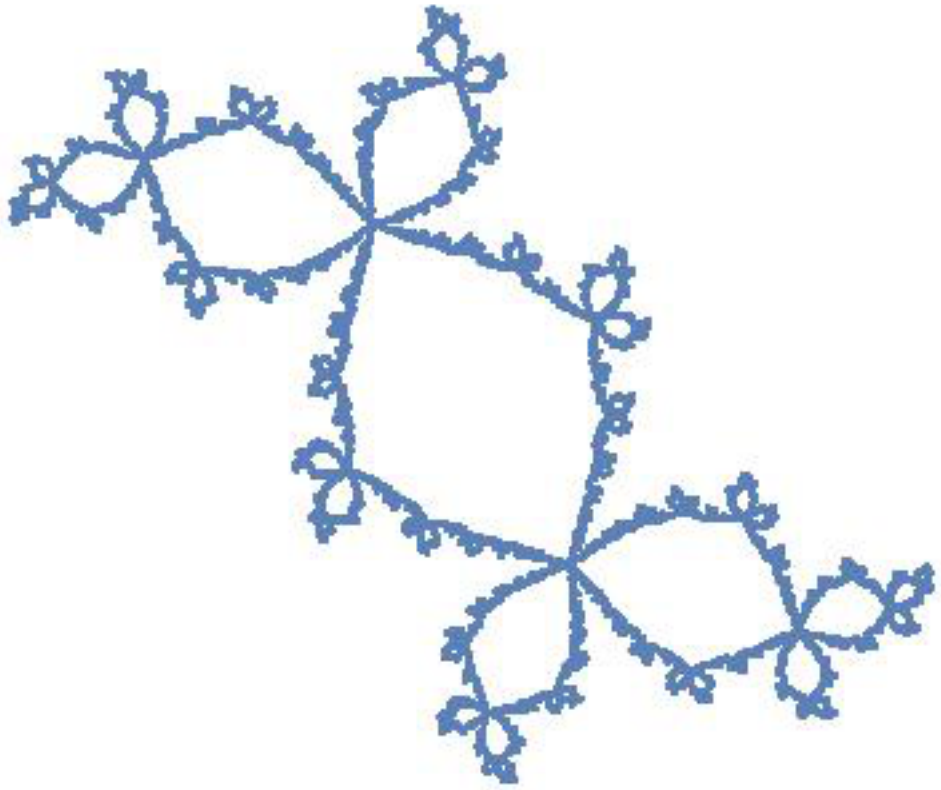

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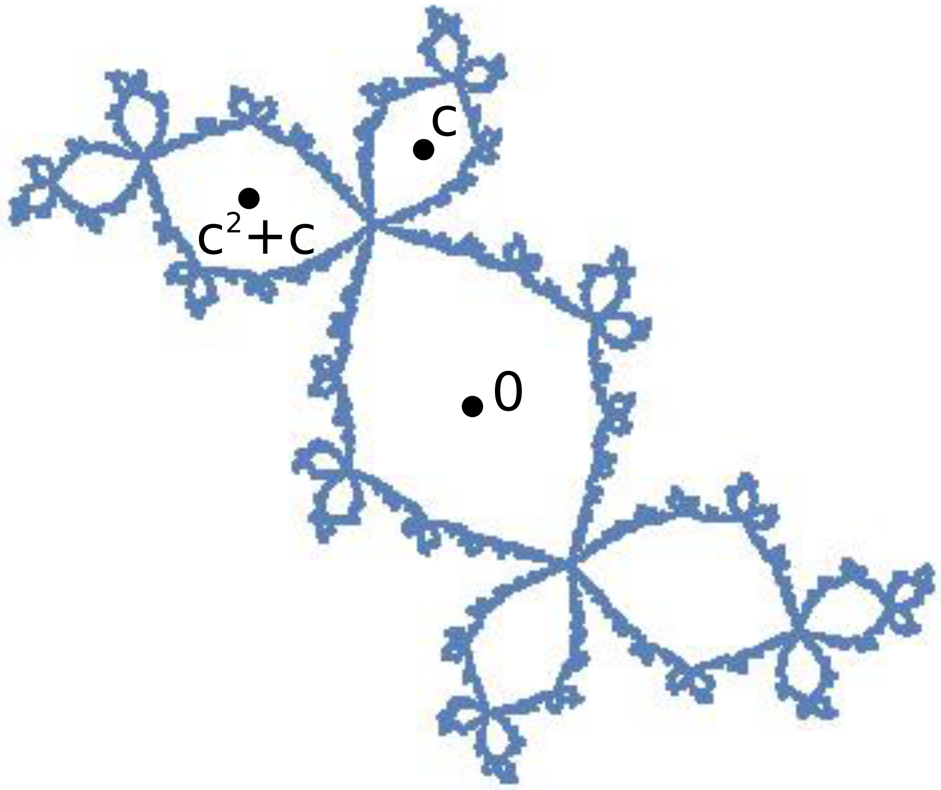

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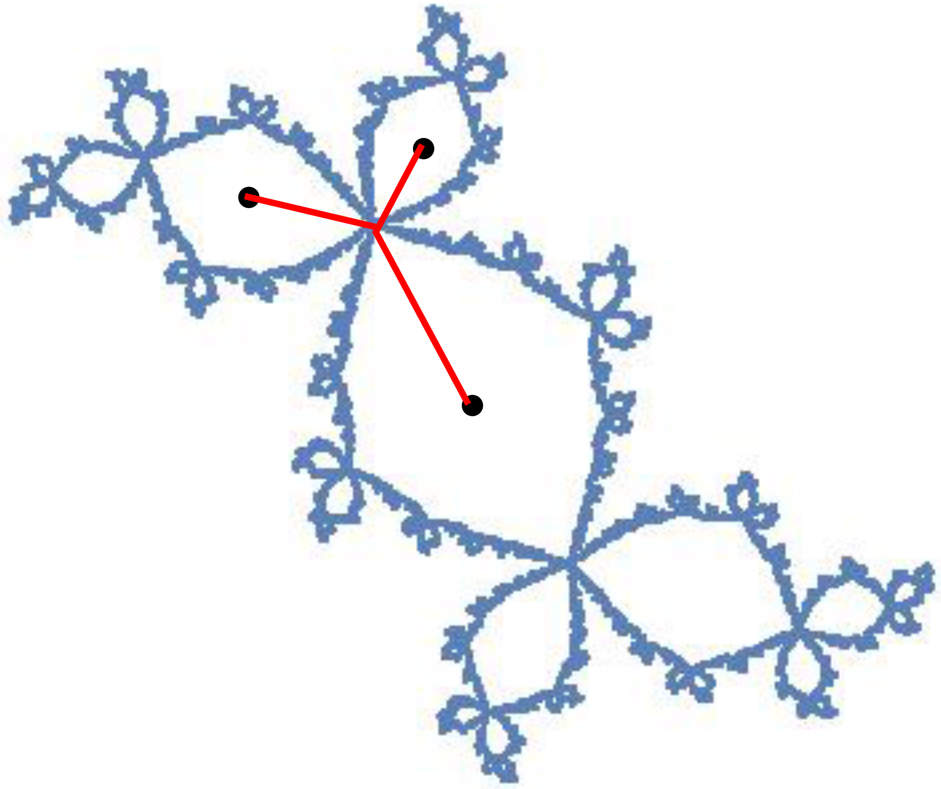

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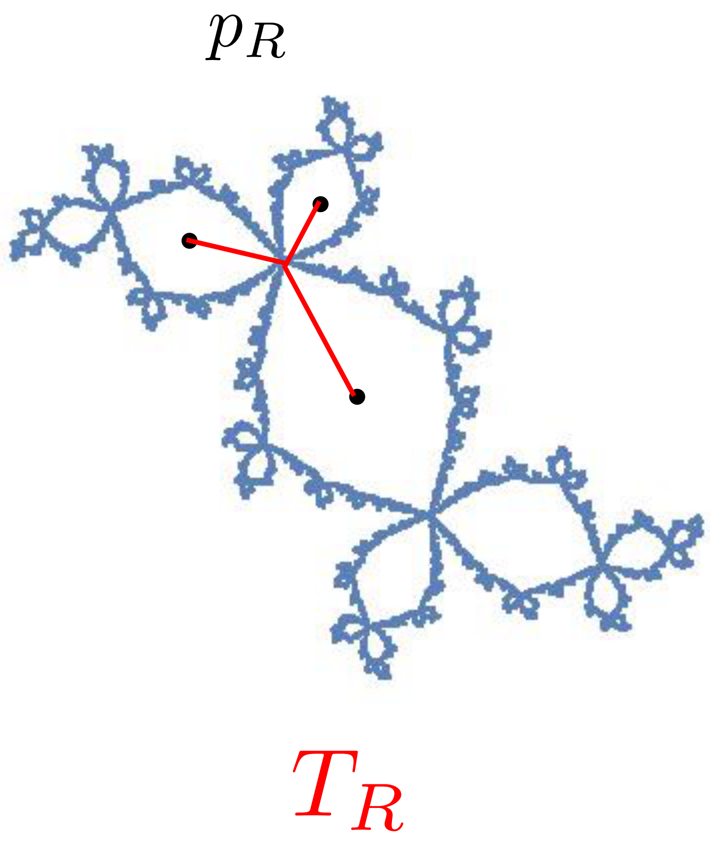

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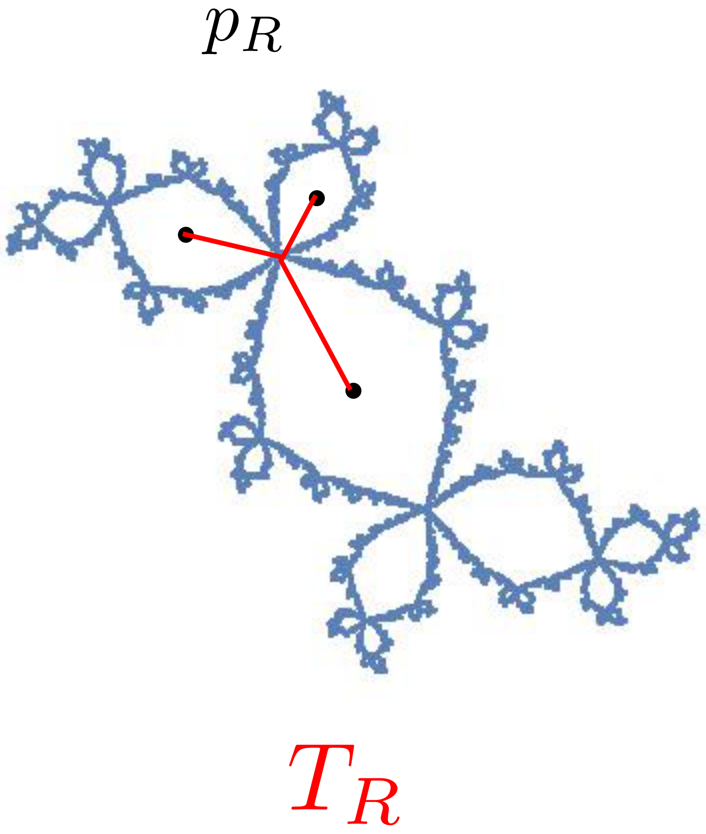

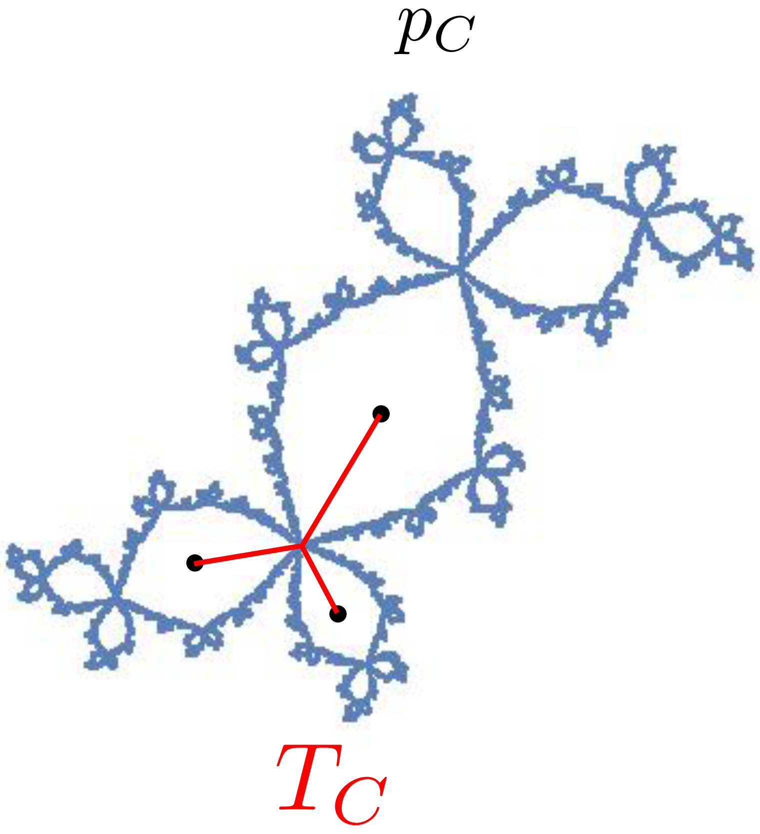

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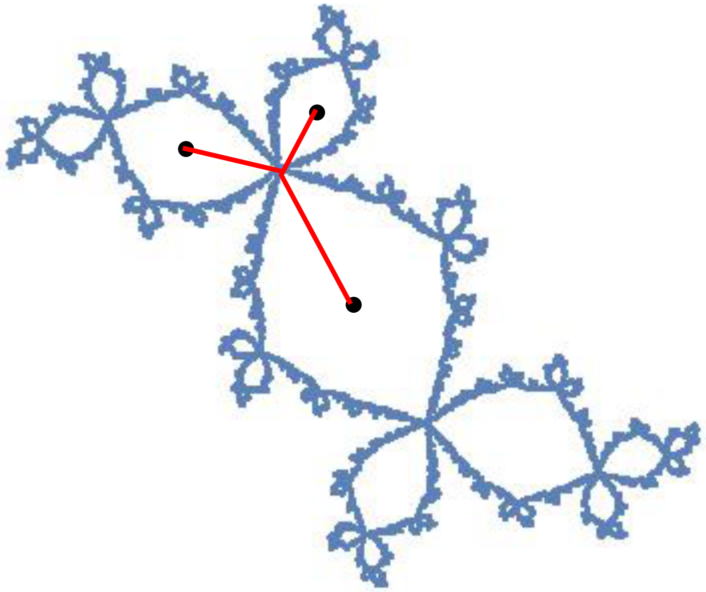
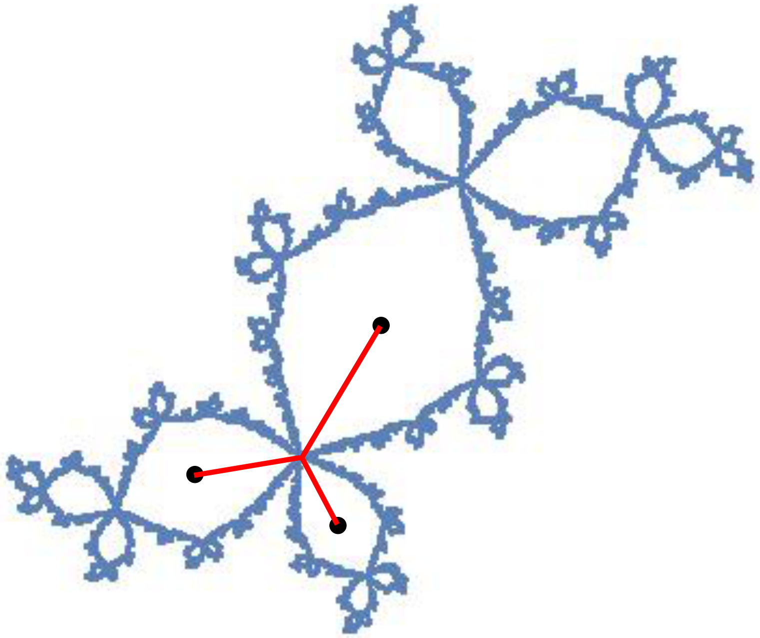


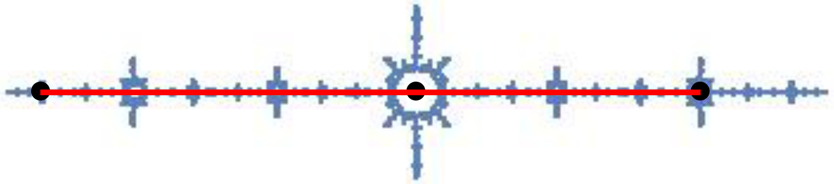
T_R

p_C



T_C

p_A



T_A

Hubbard trees as an invariant

T_A is combinatorially different from T_R and T_C

Hubbard trees as an invariant

T_A is combinatorially different from T_R and T_C

AND:

- p_R^{-1} rotates the edges of T_R clockwise
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Hubbard trees as an invariant

T_A is combinatorially different from T_R and T_C

AND:

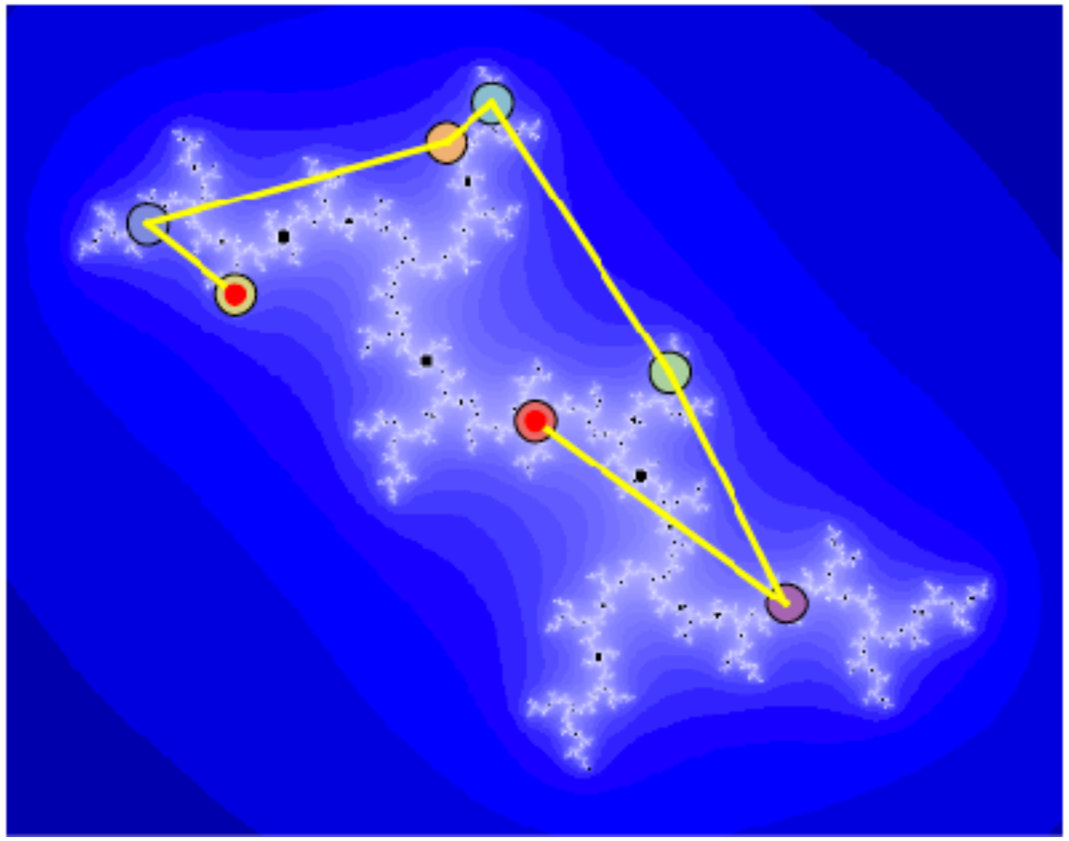
- p_R^{-1} rotates the edges of T_R clockwise
- p_C^{-1} rotates the edges of T_C counterclockwise

Proposition (Belk, Lanier, Margalit, W)
The Hubbard tree and its direction of rotation under p^{-1} distinguish p_R, p_C, p_A .

The general conjectures

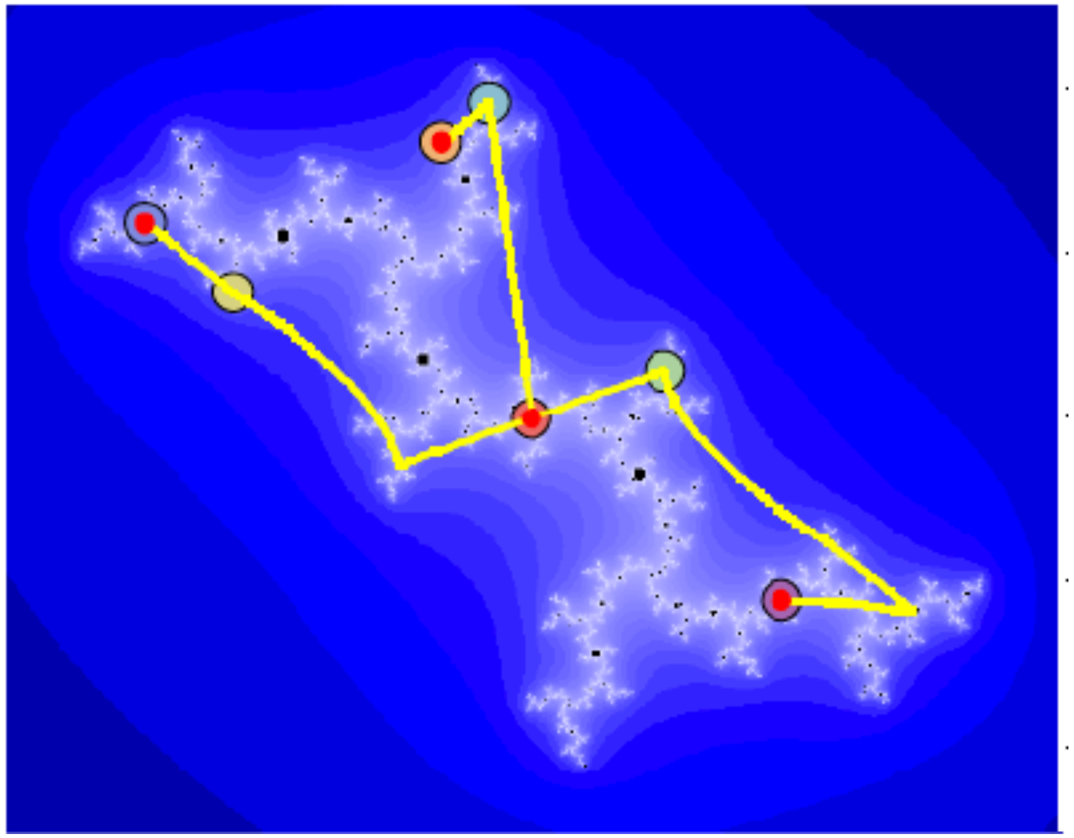
Conjecture 1: Given a polynomial p and a tree T, $\{p^{-n}(T)\}$ will converge to the Hubbard tree for p.

Tree convergence



Images by Jim Belk

Tree convergence



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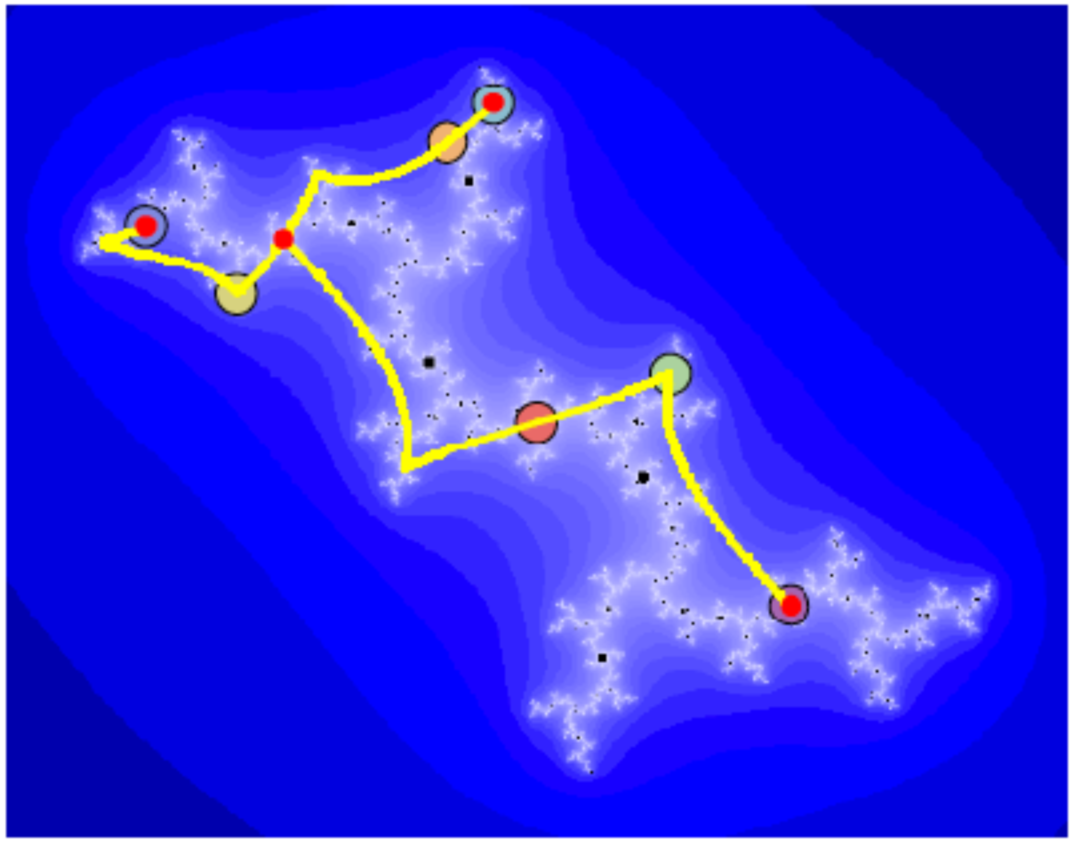

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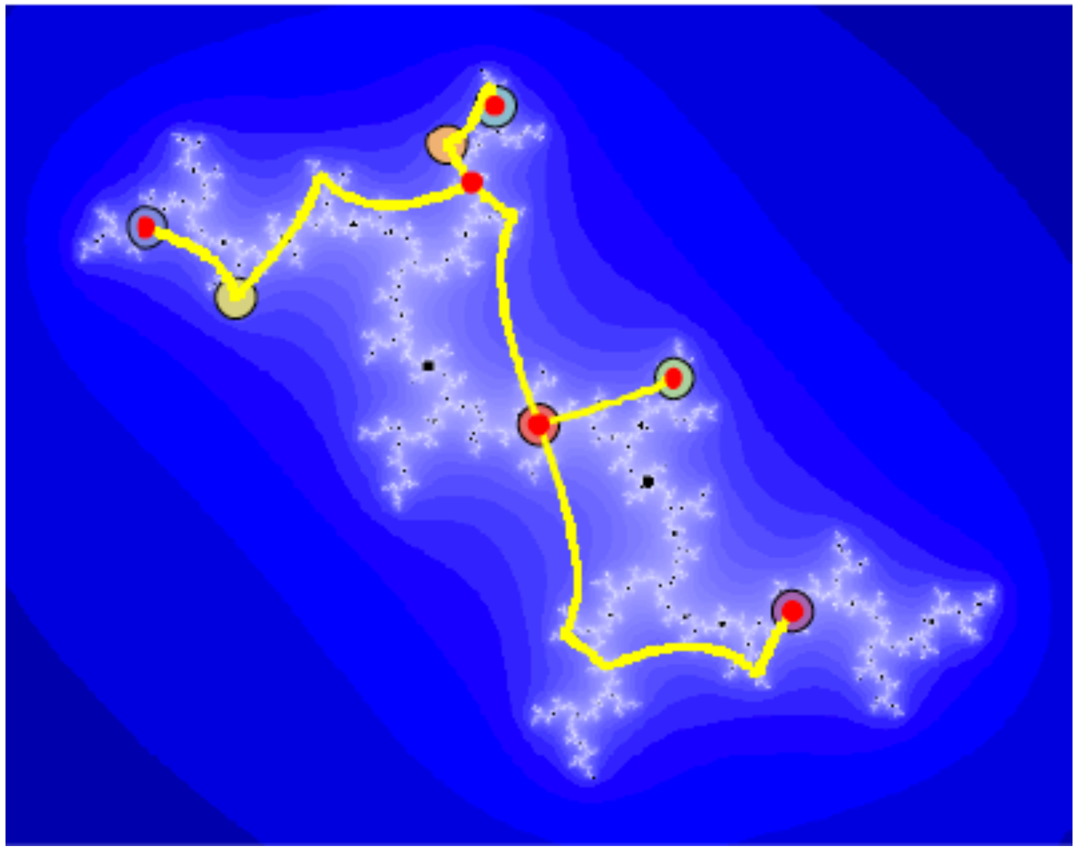

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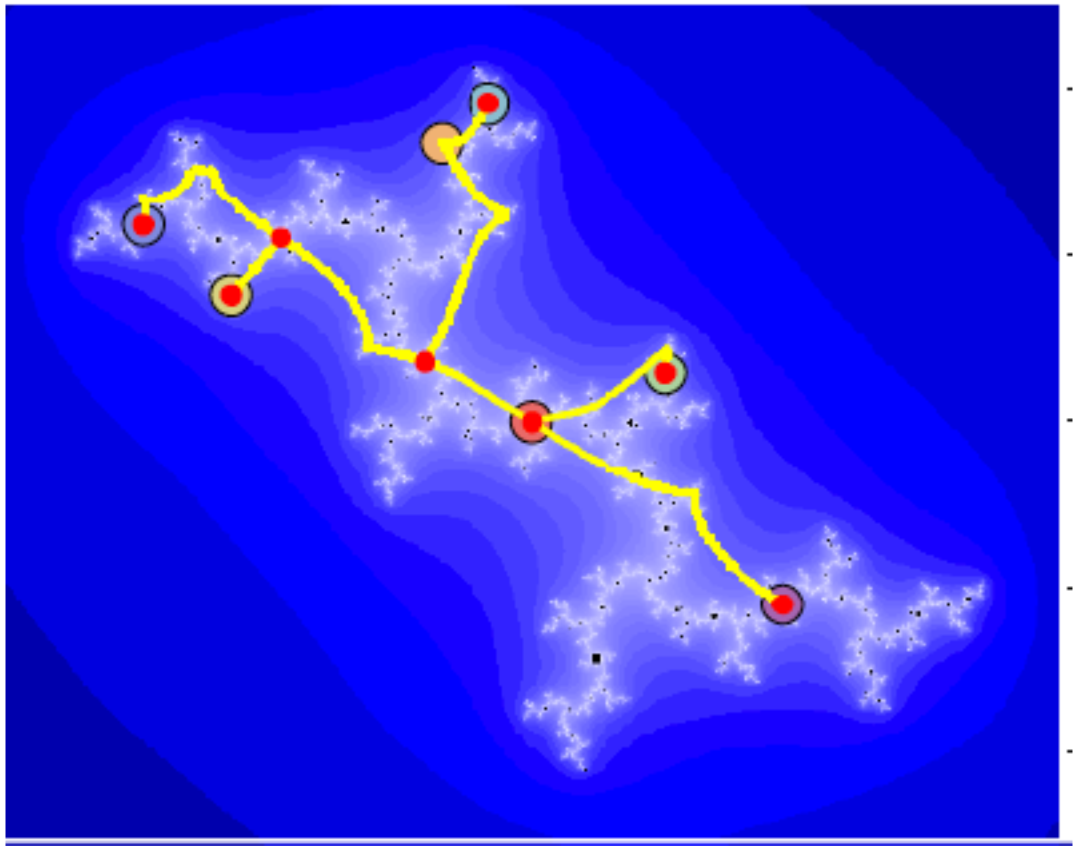

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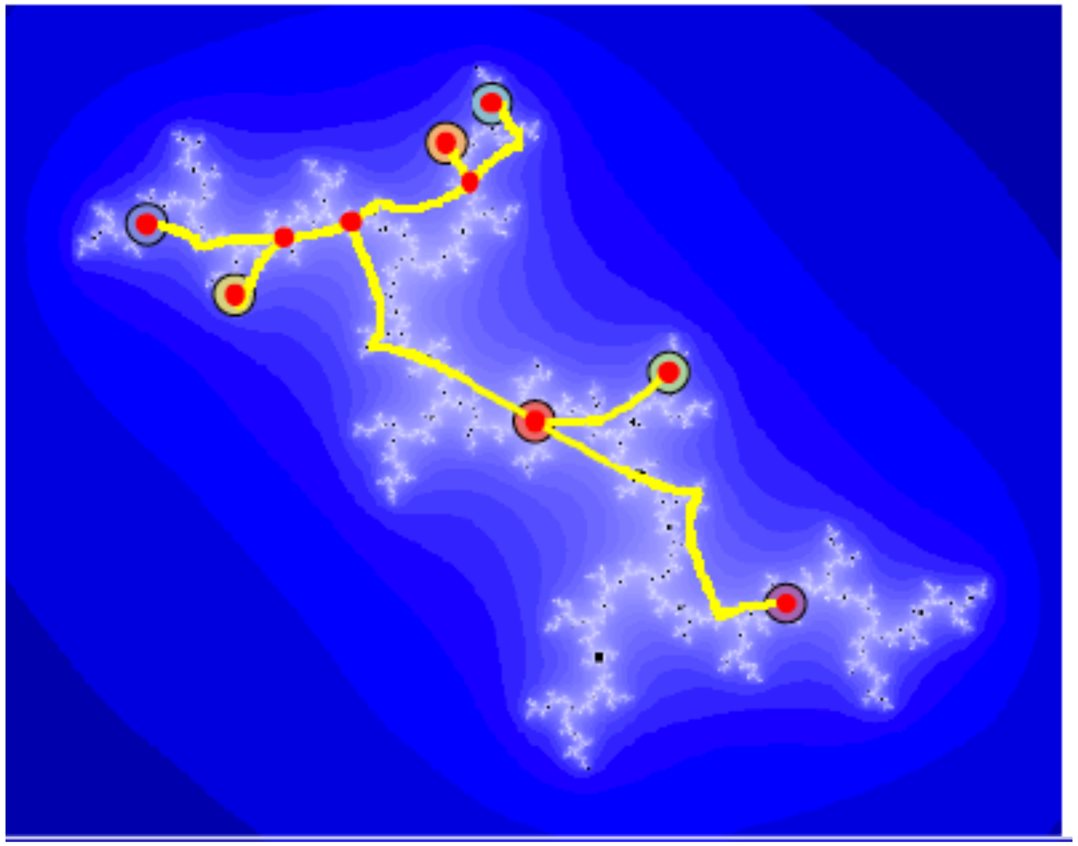

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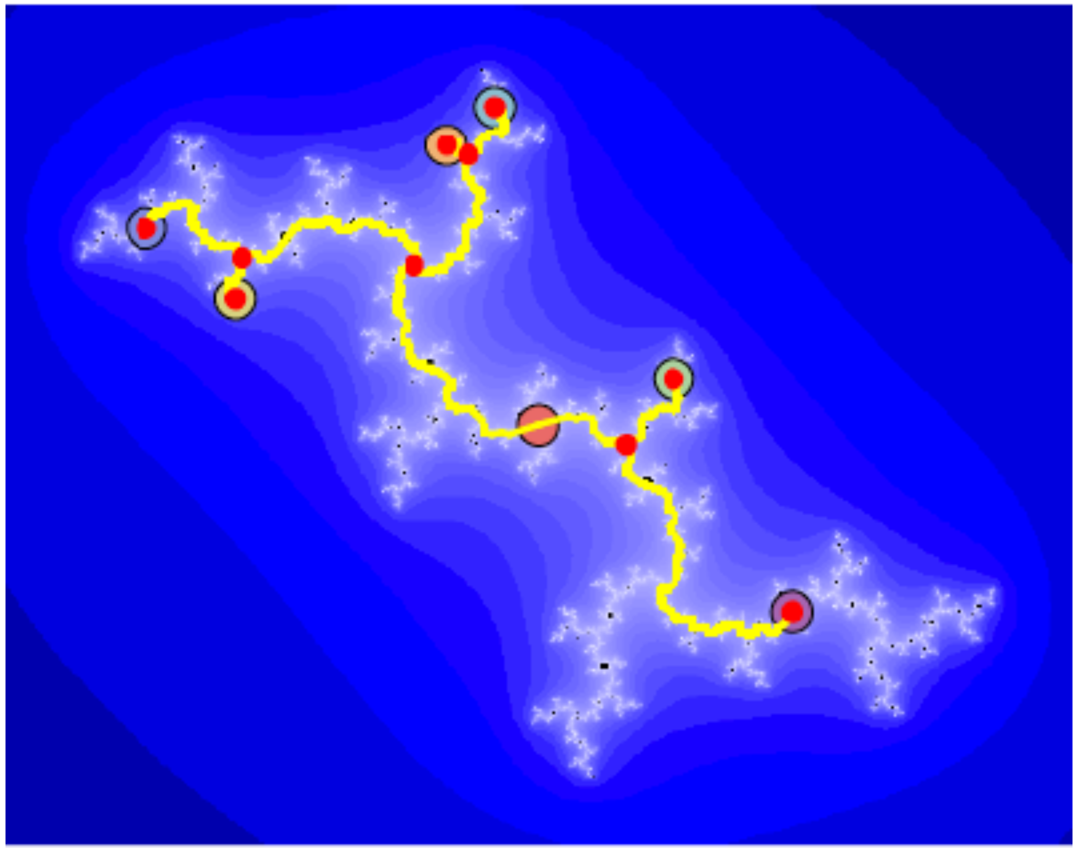

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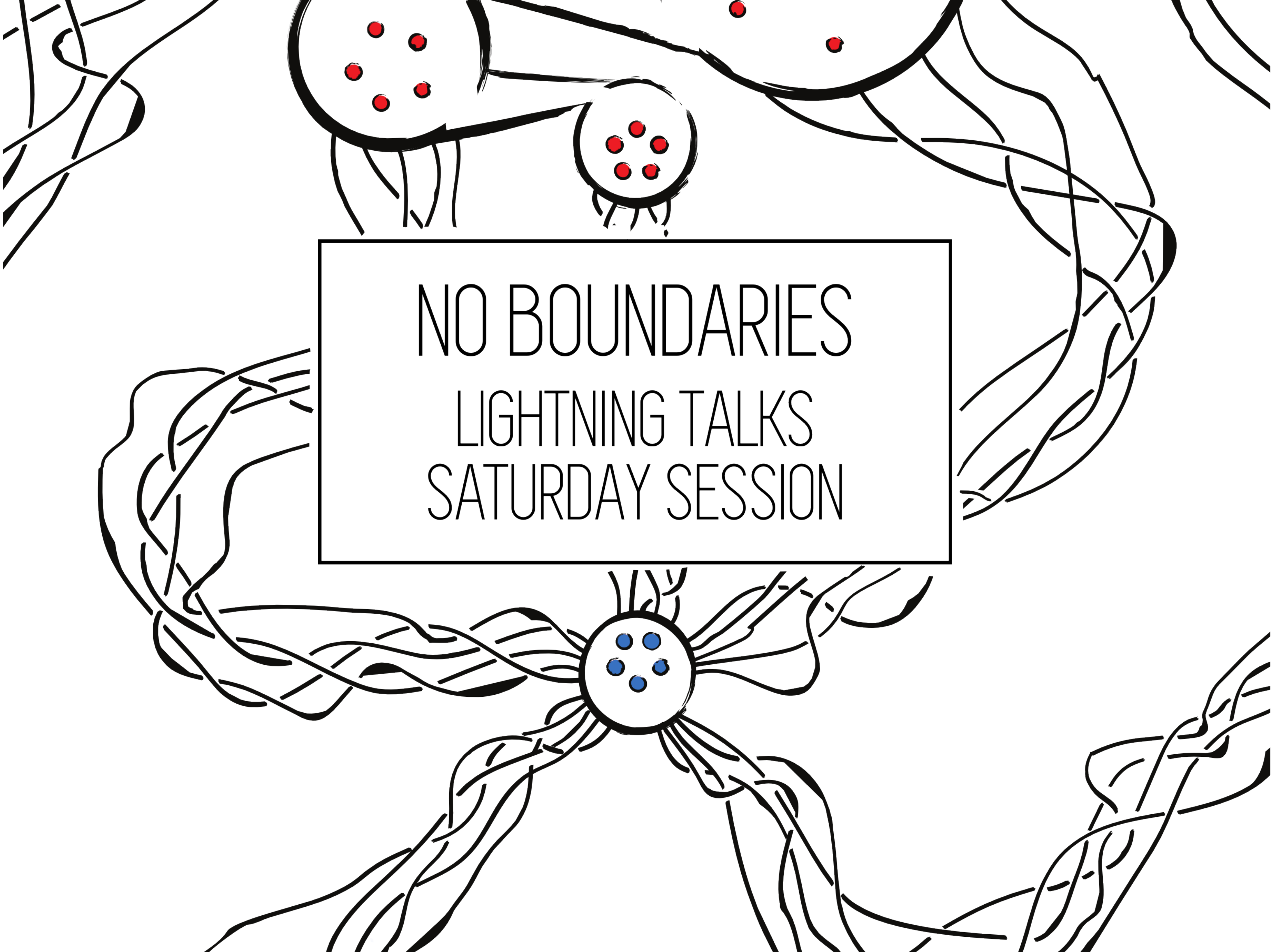
The general conjectures

Conjecture 1: Given a polynomial p and a tree T, $\{p^{-n}(T)\}$ will converge to the Hubbard tree for p.

Conjecture 2: Given polynomials p_1, p_2, the Hubbard trees and direction of rotation under p_1^{-1}, p_2^{-1} are different.



NO BOUNDARIES

LIGHTNING TALKS
SATURDAY SESSION

Homological eigenvalues of pseudo Anosov mapping classes

Asaf Hadari

University of Hawaii at Manoa

October 16, 2017

Homological representations

- Let Σ be a hyperbolic orientable surface of finite type and let $\mathrm{Mod}(\Sigma)$ be its mapping class group.

Homological representations

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- Suppose Σ has at least one puncture, or marked point. Given a finite cover $\pi : \widetilde{\Sigma} \to \Sigma$, a finite index subgroup $\Gamma < \mathrm{Mod}(\Sigma)$ lifts to $\mathrm{Mod}(\widetilde{\Sigma})$. The action of Γ on $H_1(\widetilde{\Sigma}, \mathbb{Z})$ is called the *homological representation corresponding to* π. Denote this representation ρ_π.

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- **Question:** How much information about $\mathrm{Mod}(\Sigma)$ can be recovered from its homological representations?

Images of individual elements of $\mathrm{Mod}(\Sigma)$

Given a non-identity element $f \in \mathrm{Mod}(\Sigma)$, it is easy to show that there is a cover π to which f lifts such that $\rho_\pi(f) \neq Id$. Suppose f is a pseudo Anosov mapping class. Can we recover more information about f?

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- Let $\sigma_\pi(f)$ be the spectral radius of the operator $\rho_\pi(f)$. If f has orientable stable and unstable foliations then $\sigma_\pi(f)$ is $\lambda(f)$, the dilatation of f. It is simple to show that $\sigma_\pi(f) \leq \lambda(f)$.

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- McMullen proved that $\sup_\pi \sigma_\pi(f)$ can be smaller than $\lambda(f)$.
- **Conjecture (McMullen):** For f pseudo-Anosov $\sup_\pi \sigma_\pi(f) > 1$.

Previous progress and resolution

Recently, I proved the following result which provided evidence for the McMullen conjecture.

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Theorem

Let Σ be a surface with free non-abelian fundamental group. Let $f \in \mathrm{Mod}(\Sigma)$ be an infinite order mapping class. Then there exists a cover π such that $\rho_\pi(f)$ has infinite order.

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Building on this proof, I've proved the following.

Theorem

Let Σ *be a surface with free non-abelian fundamental group. Let* $f \in \mathrm{Mod}(\Sigma)$ *be a pseudo Anosov mapping class. Then there exists a finite cover* π *such that* $\sigma_\pi(f) > 1$.

Some features of the proof

- The cover π can be taken to be characteristic, and the deck group can be taken to be solvable.

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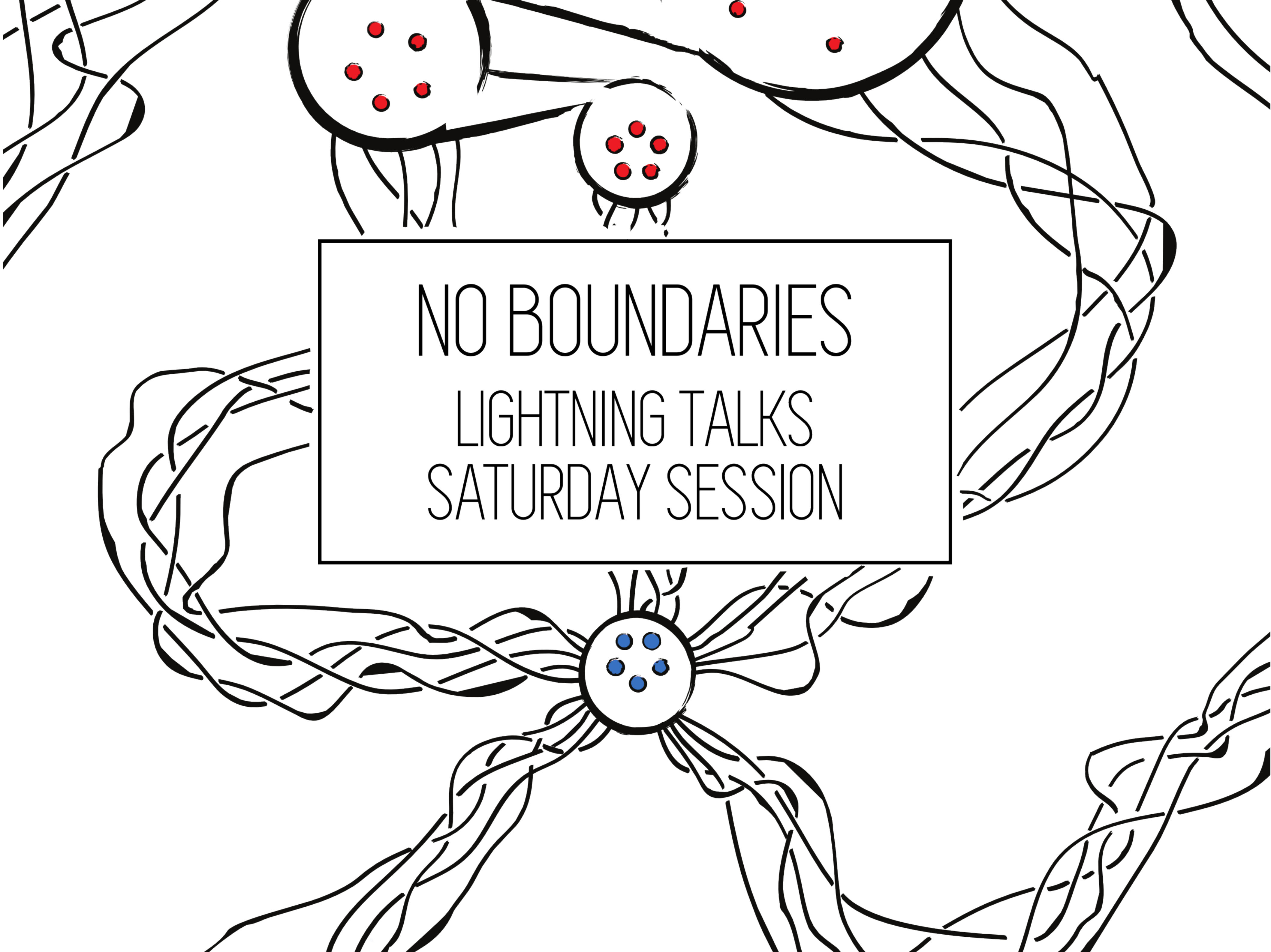
- The cover π can be taken to be characteristic, and the deck group can be taken to be solvable.
- The proof can be made to be constructive.
- The proof also works if we replace $\mathrm{Mod}(\Sigma)$ with $\mathrm{Aut}(F_n)$, and f with a fully irreducible element of $\mathrm{Aut}(F_n)$.



NO BOUNDARIES

LIGHTNING TALKS
SATURDAY SESSION

Mapping class groups and monodromy of families of plane curves

Nick Salter
Harvard University
October 30, 2017

Plane curves

Plane curves

Object of study:

Plane curves

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E.g. $\{[X : Y : Z] \in \mathbb{C}P^2 \mid X^d + Y^d + Z^d = 0\}$

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Monodromy: Homomorphism $\rho_d : \pi_1(\mathcal{P}_d) \to \mathrm{Mod}(\Sigma_g)$

Basic question: What is $\Gamma_d := \mathrm{im}(\rho_d)$?

What is known?

What is known?

Beauville:

What is known?

Beauville: Computation of “cohomological monodromy”

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Key points:

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For d even, *surjective*

For d odd, must preserve a *spin structure*

What is known?

Beauville: Computation of “cohomological monodromy”

Key points: Always finite-index in $\mathrm{Sp}(2g, \mathbb{Z})$

For d even, *surjective*

For d odd, must preserve a *spin structure*

Question: Are there “non-cohomological” obstructions?

Does $\Gamma_d = \mathrm{Mod}(\Sigma_g)$ for d even?

Some prior work

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Folklore observation:

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Define $\mathrm{Mod}(\Sigma_g)[\phi_d]$ as the stabilizer of ϕ_d (a higher spin structure)

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Theorem (S., ’16): $\Gamma_5 = \mathrm{Mod}(\Sigma_6)[\phi_5]$

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Theorem (S., ’16): $\Gamma_5 = \mathrm{Mod}(\Sigma_6)[\phi_5]$

Method: reduce to Johnson’s work on Torelli group

New theorem

New theorem

Theorem (S., Tuesday):

New theorem

Theorem (S., Tuesday): $\Gamma_d = \mathrm{Mod}(\Sigma_g)[\phi_d]$ for all d even

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Points of interest:

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Points of interest:

Determine generators for $\mathrm{Mod}(\Sigma_g)[\phi_d]$

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Tip of an iceberg: line bundles on toric varieties

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(results also for curves in $\mathbb{C}P^1 \times \mathbb{C}P^1$, Hirzebruch surfaces, etc.)

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Uses tropical geometry methods developed by Crétois-Lang

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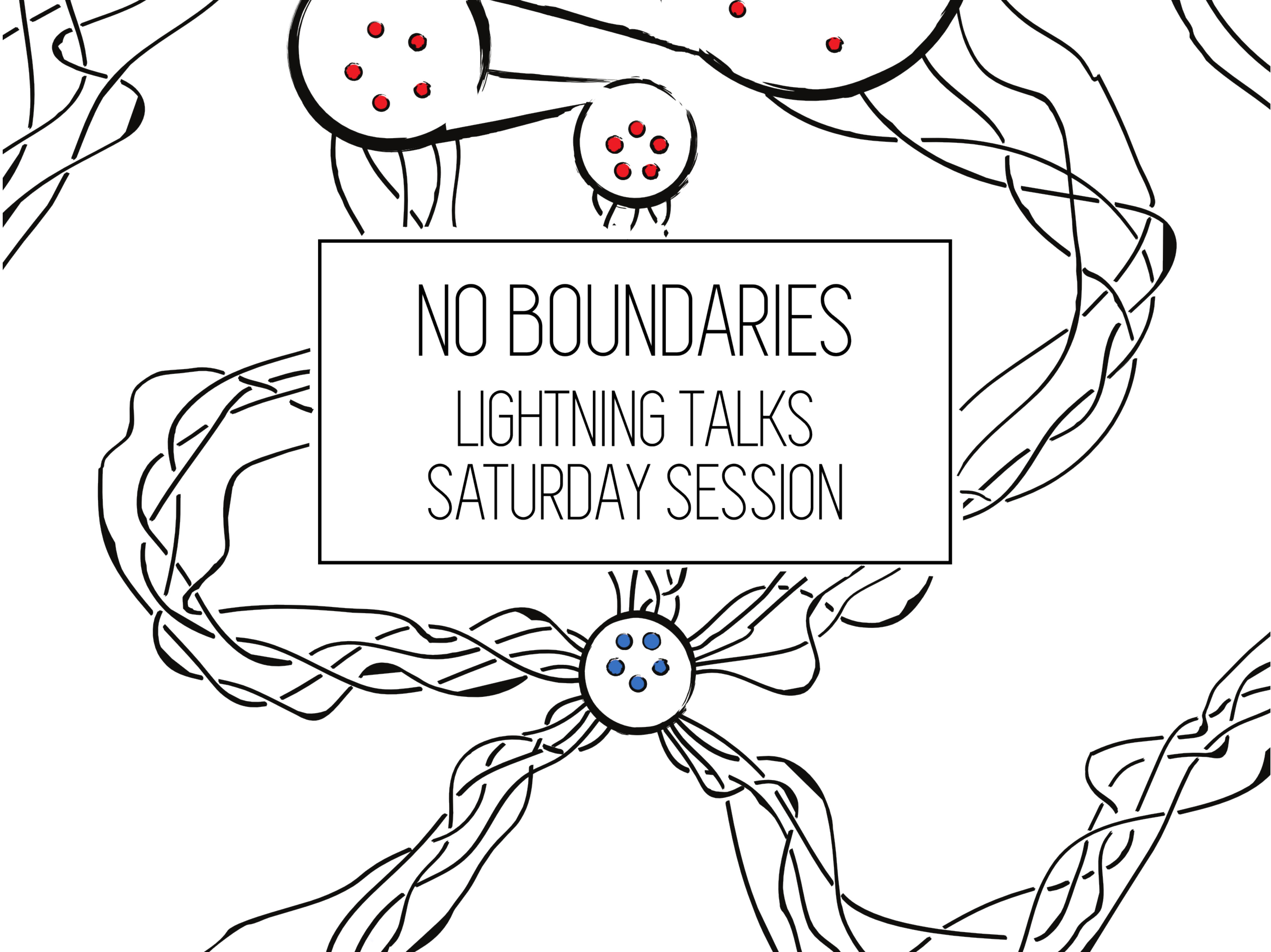
Answers a question of Donaldson from 2000



NO BOUNDARIES

LIGHTNING TALKS
SATURDAY SESSION

Semidualities from products of trees

Daniel Studenmund
joint with Kevin Wortman

University of Notre Dame

No Boundaries, October 2017

Theorem (Borel–Serre)

Suppose G is a semisimple algebraic group defined over $\mathbb{Q}$ and $\Gamma \leq G$ an arithmetic subgroup. Then Γ is a $\mathbb{Q}$-duality group*: there is a number d such that for any $\mathbb{Q}\Gamma$-module M there are isomorphisms*

$$H^k(\Gamma, M) \cong H_{d-k}(\Gamma, D \otimes_{\mathbb{Q}} M),$$

where $D = H^d(\Gamma, \mathbb{Q}\Gamma)$.

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where $D = H^d(\Gamma, \mathbb{Q}\Gamma)$.

If Γ is cocompact in $G(\mathbb{R})$, then $D = \mathbb{Q}$.

Theorem (Borel–Serre)

*Suppose G is a semisimple algebraic group defined over $\mathbb{Q}$ and $\Gamma \leq G$ an arithmetic subgroup. Then Γ is a $\mathbb{Q}$-*duality group*: there is a number d such that for any $\mathbb{Q}\Gamma$-module M there are isomorphisms*

$$H^k(\Gamma, M) \cong H_{d-k}(\Gamma, D \otimes_{\mathbb{Q}} M),$$

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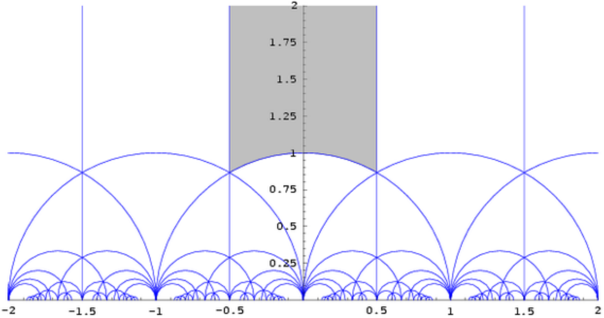
Example: If $\Gamma = \mathrm{SL}_2(\mathbb{Z})$ then $d = 1$ and $D = H^1(\Gamma, \mathbb{Q}\Gamma) \cong \bigoplus_{P^1(\mathbb{Q})} \mathbb{Q}$

- $\Gamma = \mathrm{SL}_2(\mathbb{Z})$ acts on hyperbolic plane $\mathbb{H}^2$:



https://golem.ph.utexas.edu/category/2008/02/modular_forms.html

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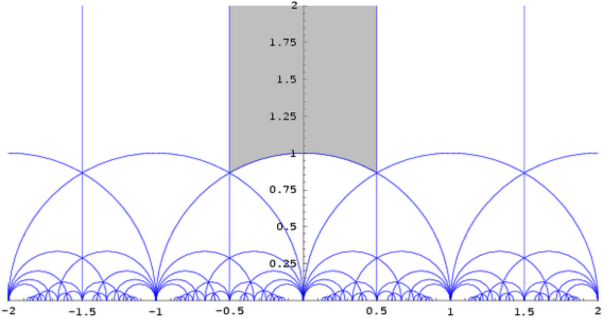

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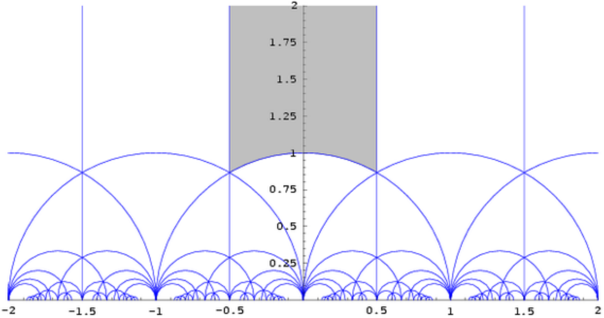

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- Γ acts cocompactly on $\widehat{\mathbb{H}^2} = \mathbb{H}^2 - \bigcup_{x \in P^1(\mathbb{Q})} B_x$, for horoballs B_x
- $H^1(\Gamma, \mathbb{Q}\Gamma) \cong H^1_c(\widehat{\mathbb{H}^2}) \cong H_0(\widehat{\mathbb{H}^2}, \partial\widehat{\mathbb{H}^2}) \cong \tilde{H}_0(\partial\widehat{\mathbb{H}^2}) \cong \tilde{H}_0(P^1(\mathbb{Q}))$

Fact: Γ is a $\mathbb{Q}$-duality group of dimension d iff

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Theorem: If Γ is a semiduality group then for any M there are maps

$$\phi : H_{d-k}(\Gamma, D \otimes_{\mathbb{Q}} M) \to H^k(\Gamma, M)$$

that are isomorphisms for sufficiently 'nice' M.

Conjecture: Suppose G is a simple algebraic group defined over a global function field K of characteristic p and Γ is an S-arithmetic subgroup. Then Γ is a $\mathbb{Q}$-semiduality group.

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Example: $K = \mathbb{F}_2(t)$ and $\Gamma = SL_2(\mathbb{F}_2[t, t^{-1}])$

Compute $H^n(\Gamma, \mathbb{Q}\Gamma)$ for $\Gamma = \mathrm{SL}_2(\mathbb{F}_2[t, t^{-1}]) \curvearrowright T \times T$

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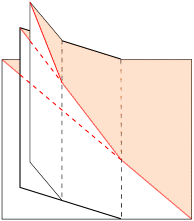
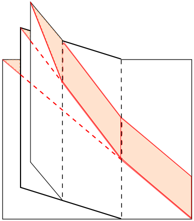
Horoball $B =$  filtered by $B_n =$ 

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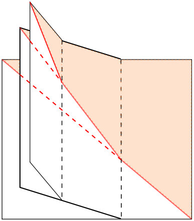

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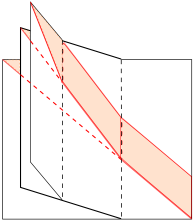

Dualizing module satisfies:

$$0 \to H^2_c(T \times T) \to H^2(\Gamma, \mathbb{Q}\Gamma) \to \bigoplus_{x \in P^1(K)} \varprojlim{}^1 H^1_c(B_n) \to 0$$

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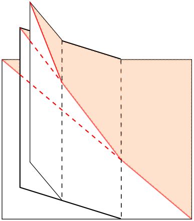

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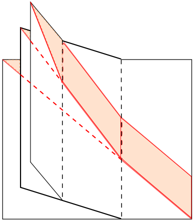

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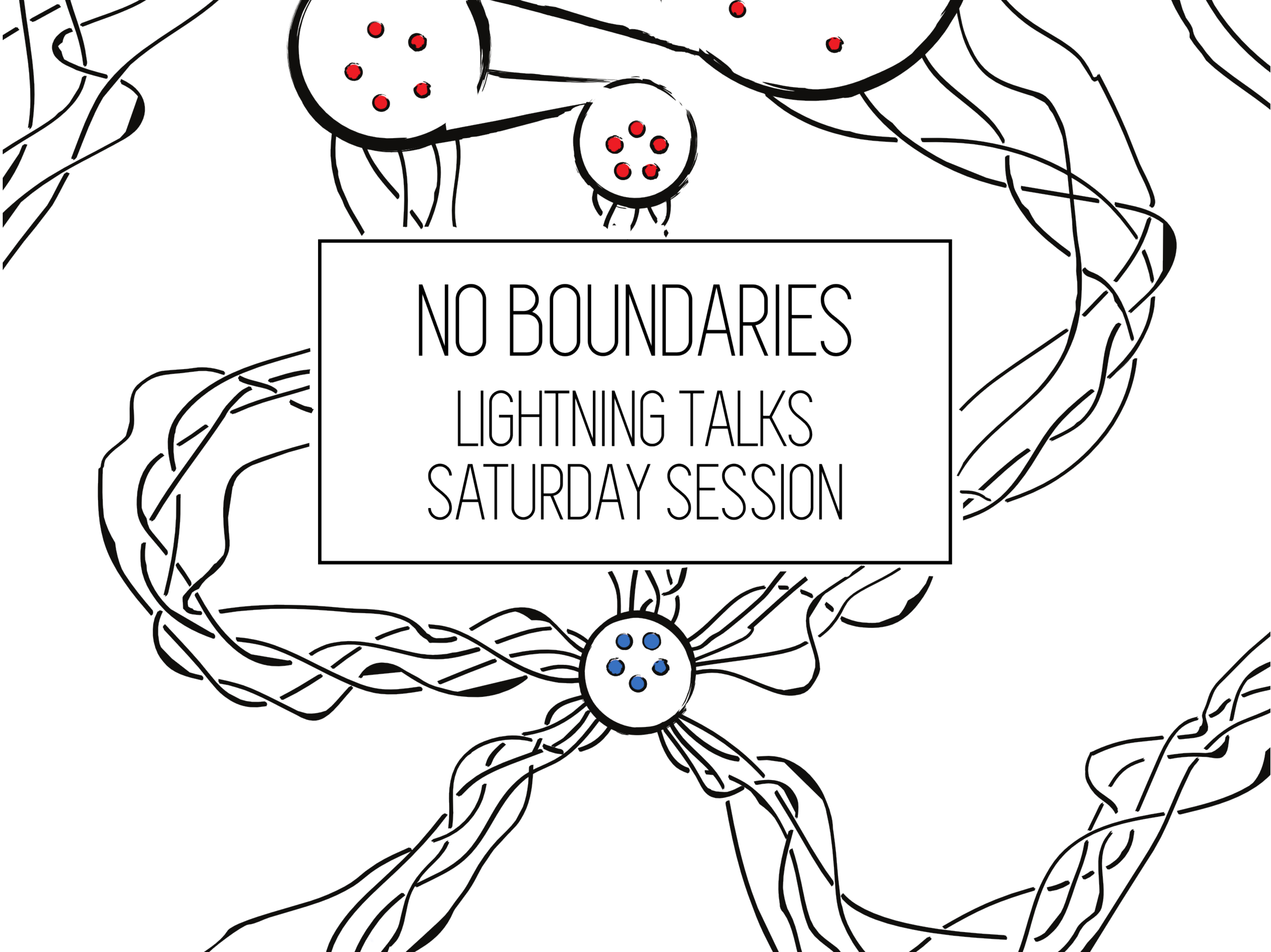
Thank you! Happy birthday, Benson!



NO BOUNDARIES

LIGHTNING TALKS
SATURDAY SESSION

Fast Nielsen-Thurston Classification

Balázs Strenner

Georgia Institute of Technology

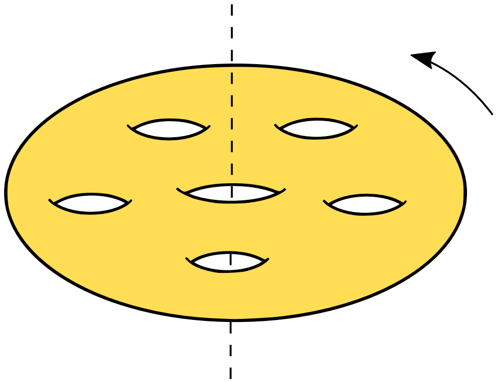
joint with Dan Margalit and S. Öykü Yurttaş

No boundaries – Groups in algebra, geometry and topology
University of Chicago
October 28, 2017

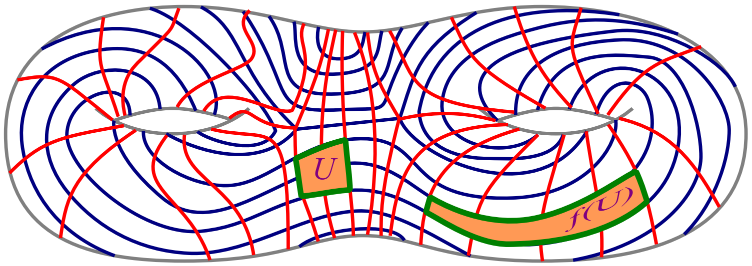
Benson

The Nielsen-Thurston Classification

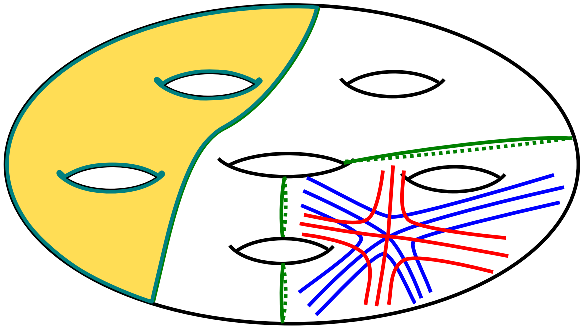
Mod(S)



Finite order



Pseudo-Anosov



Reducible

The Nielsen-Thurston Classification Problem

Fix finite generating set for Mod(S).

OUTPUT

INPUT

$f = s_1s_2...s_n$

Algorithm

Finite order

order

Reducible

reducing curves

Pseudo-Anosov

stretch factor

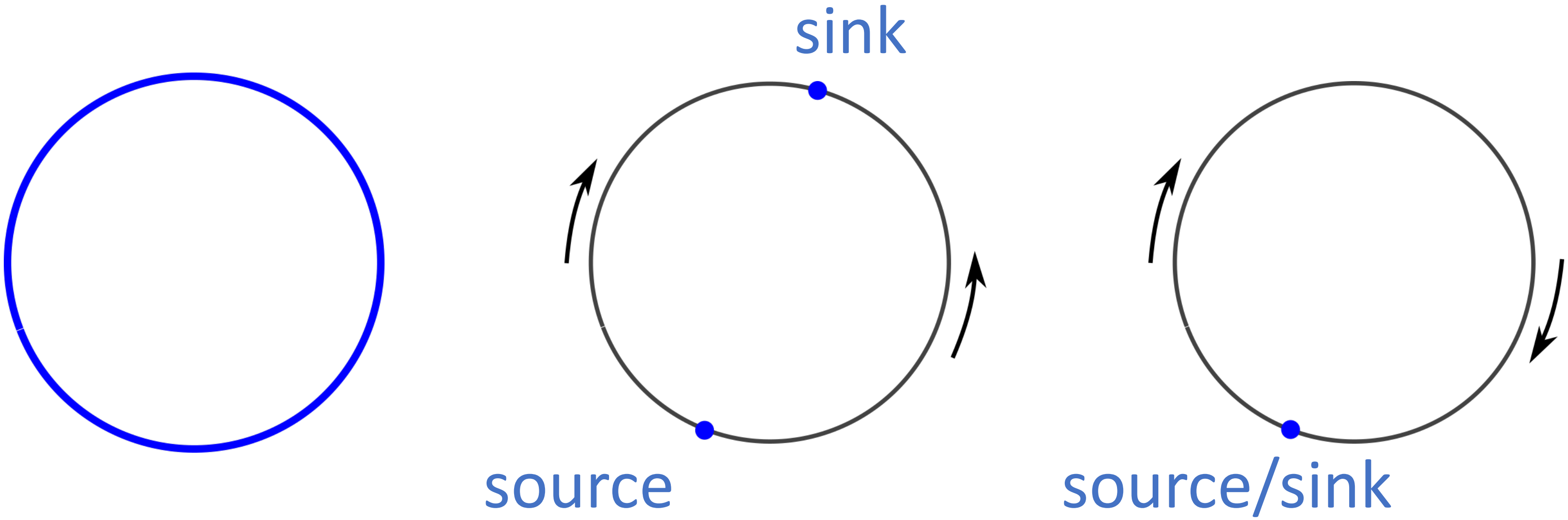
foliations

Running time: function of |f|.

Main Theorem

Theorem (Margalit-S-Yurttaş): There exists a quadratic-time algorithm for the Nielsen-Thurston Classification Problem.

Isometries of $\mathbb{H}^2$



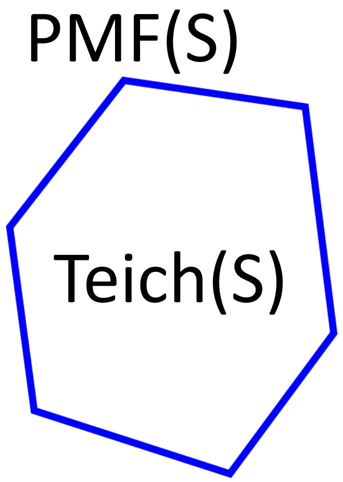
elliptic

(up to power)

hyperbolic

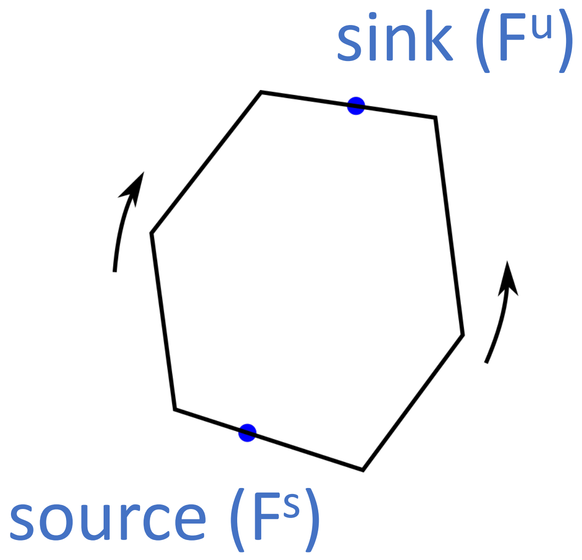
parabolic

The mapping class group

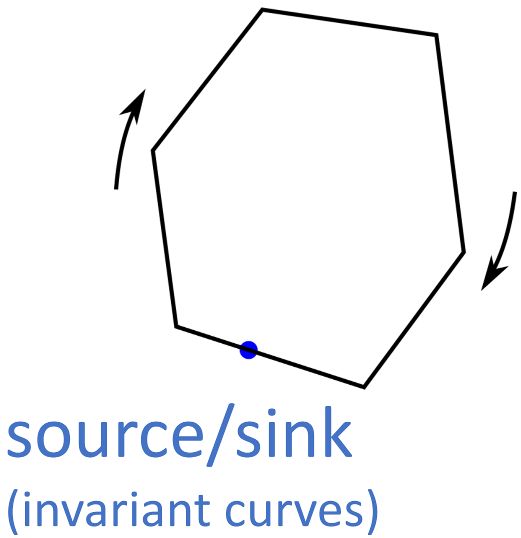


finite order

(up to power)

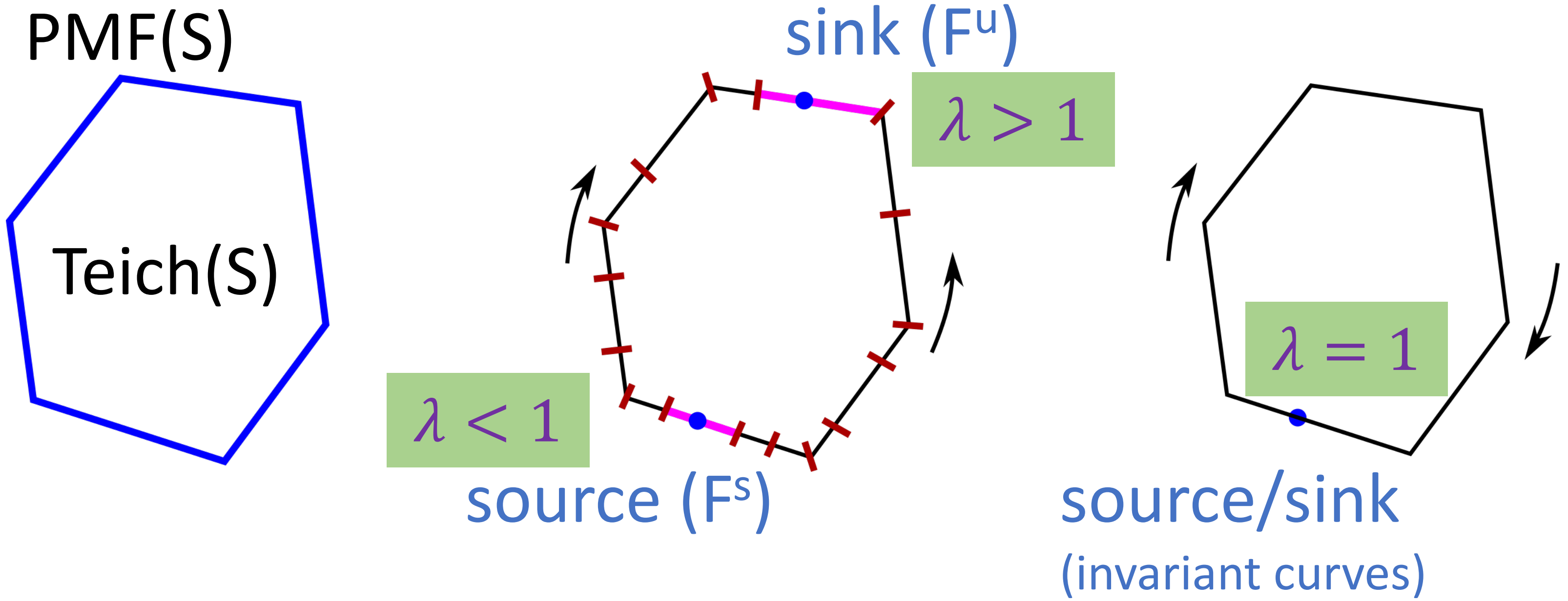


pseudo-Anosov



Reducible

Thurston, Mosher: Compute the piecewise linear map and find all the eigenvectors.



finite order
(up to power)

pseudo-Anosov

Reducible

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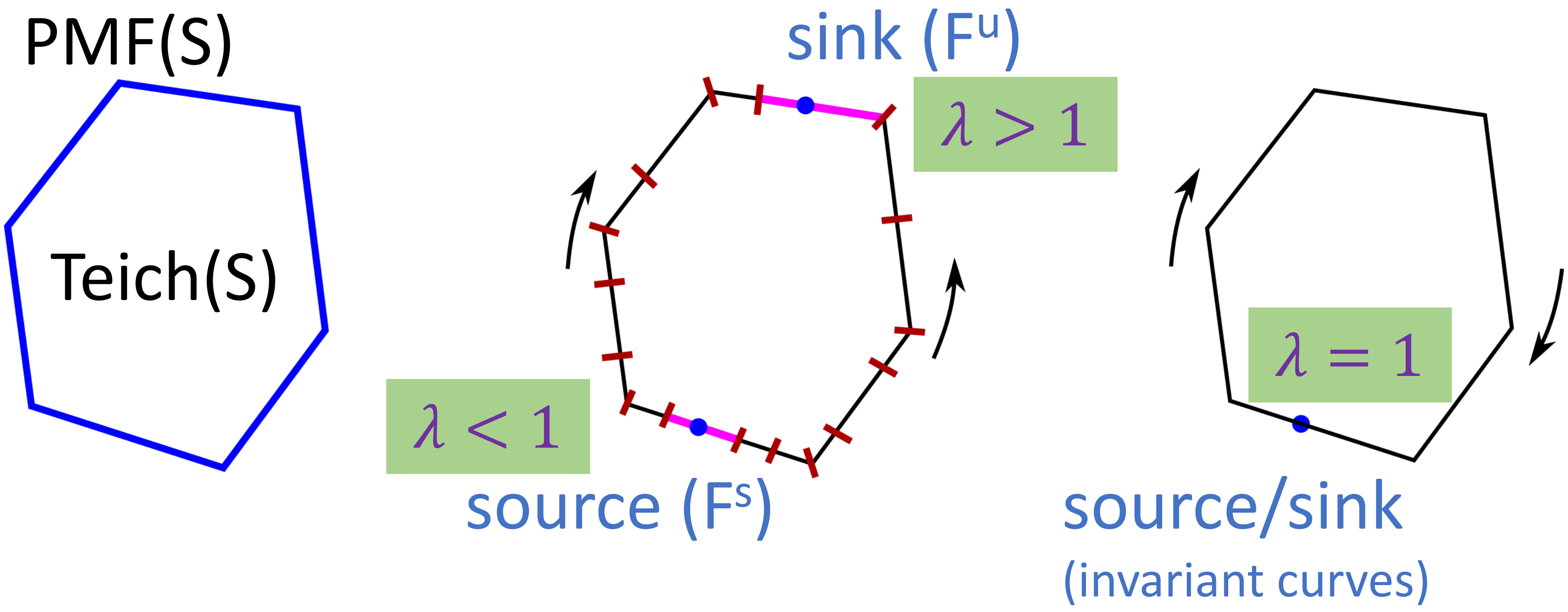
Exponentially many pieces ☹️



PMF(S)

Teich(S)

sink (F^u)

$\lambda > 1$

$\lambda < 1$

source (F^s)

$\lambda = 1$

source/sink

(invariant curves)

finite order

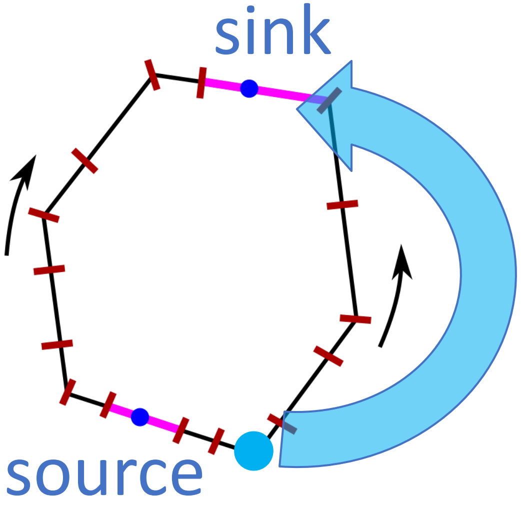
(up to power)

pseudo-Anosov

Reducible

Iterate!

Toby Hall (Dynn): First iterate, then compute eigenvectors. 😀



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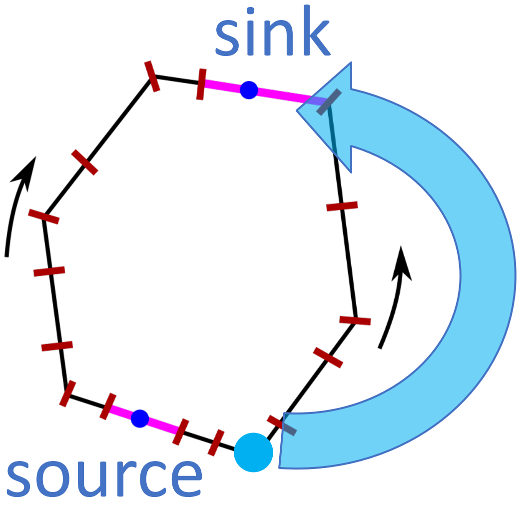
Unknown behavior in reducible case 😩



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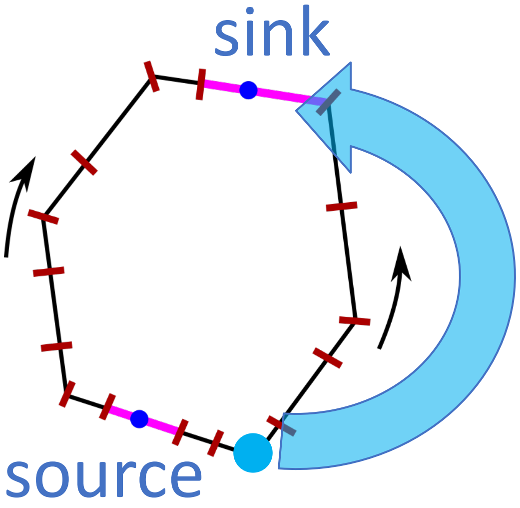

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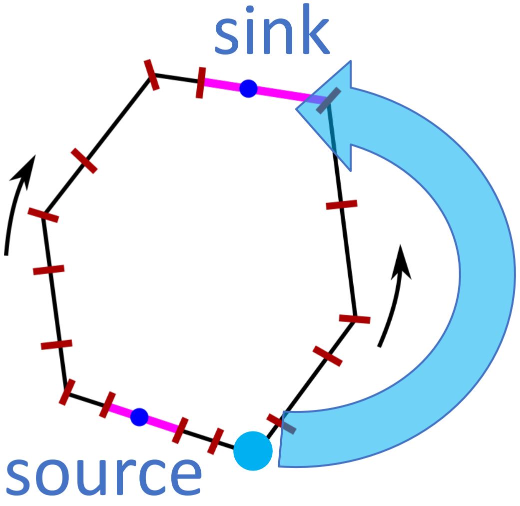
Margalit-S-Yurttaş: O(1) iterations is enough. 👍🏼



O(1) iterations

Macaw (implementation)

1. ***Works for closed surfaces***
2. Solves the word problem
3. Approximates stretch factors
4. Computes the order

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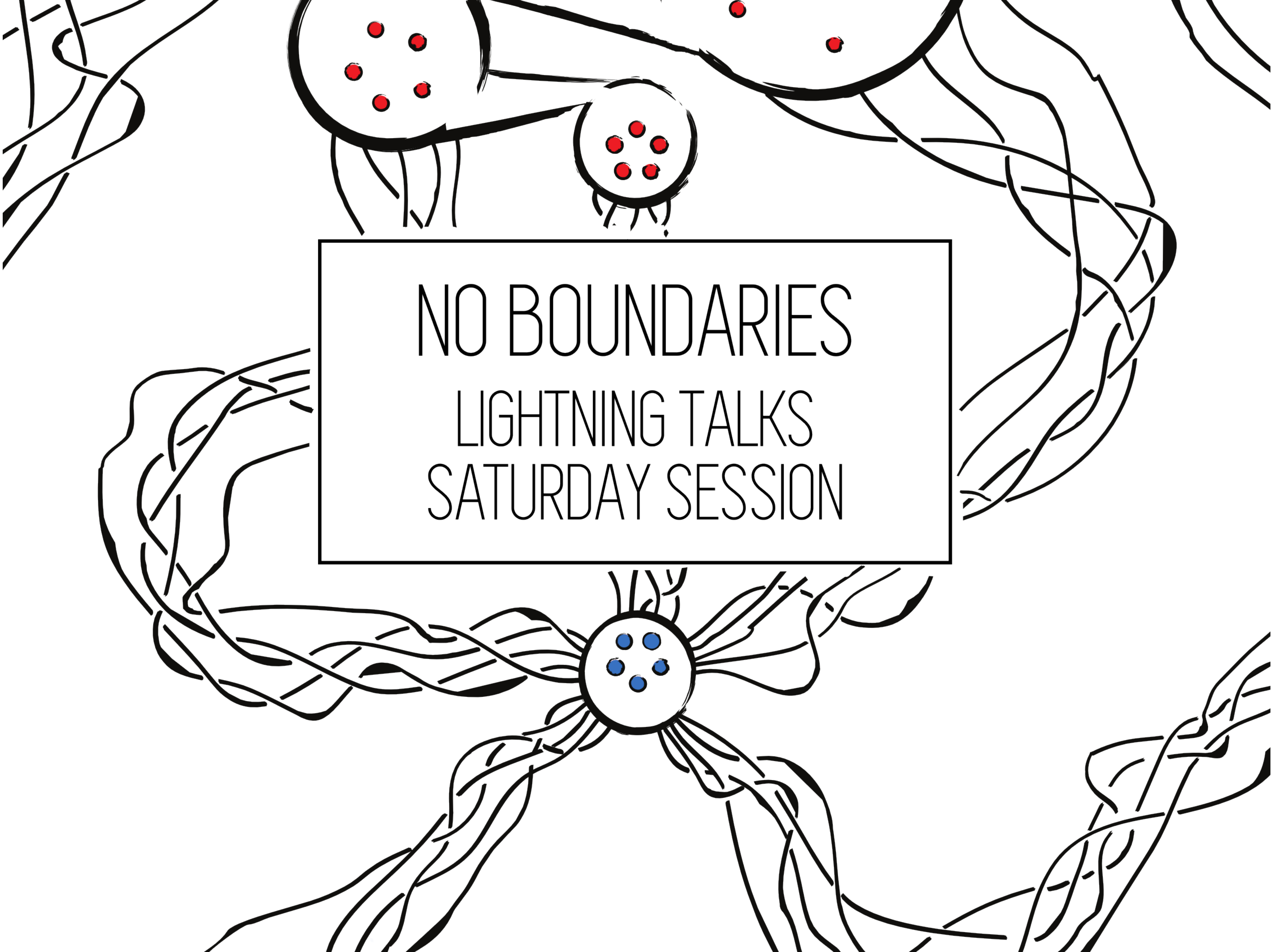

NO BOUNDARIES

LIGHTNING TALKS
SATURDAY SESSION

Adding points to configurations

Lei Chen
University of Chicago

The general problem

The general problem

$$\begin{tikzcd}
\mathrm{Conf}_{n+1}(M) \arrow[d, "\text{Forget}"'] \\
\mathrm{Conf}_{n}(M)
\end{tikzcd}$$

The general problem

$$\begin{tikzcd}
\mathrm{Conf}_{n+1}(M) \arrow[d, shift right=2, "\text{Forget}"'] \\
\mathrm{Conf}_n(M) \arrow[u, shift right=2, "?"']
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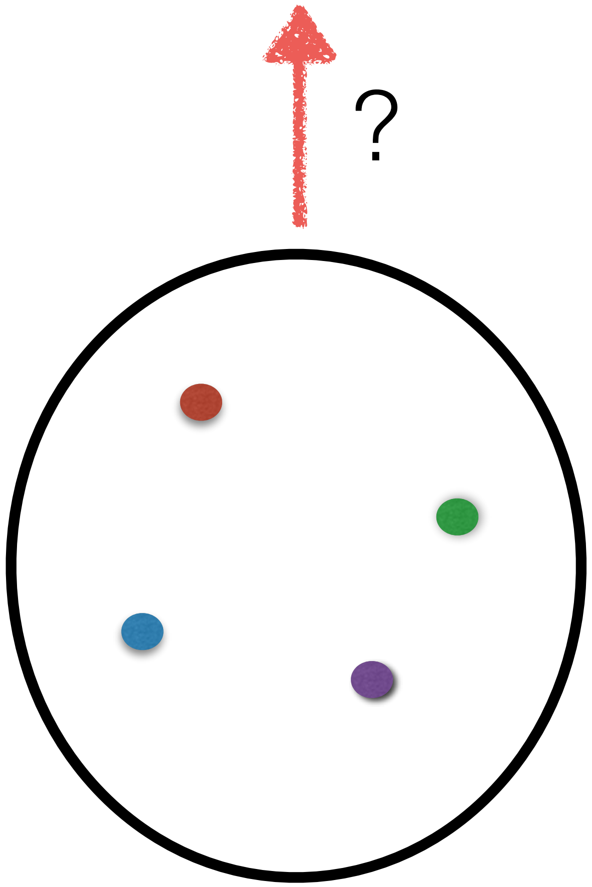

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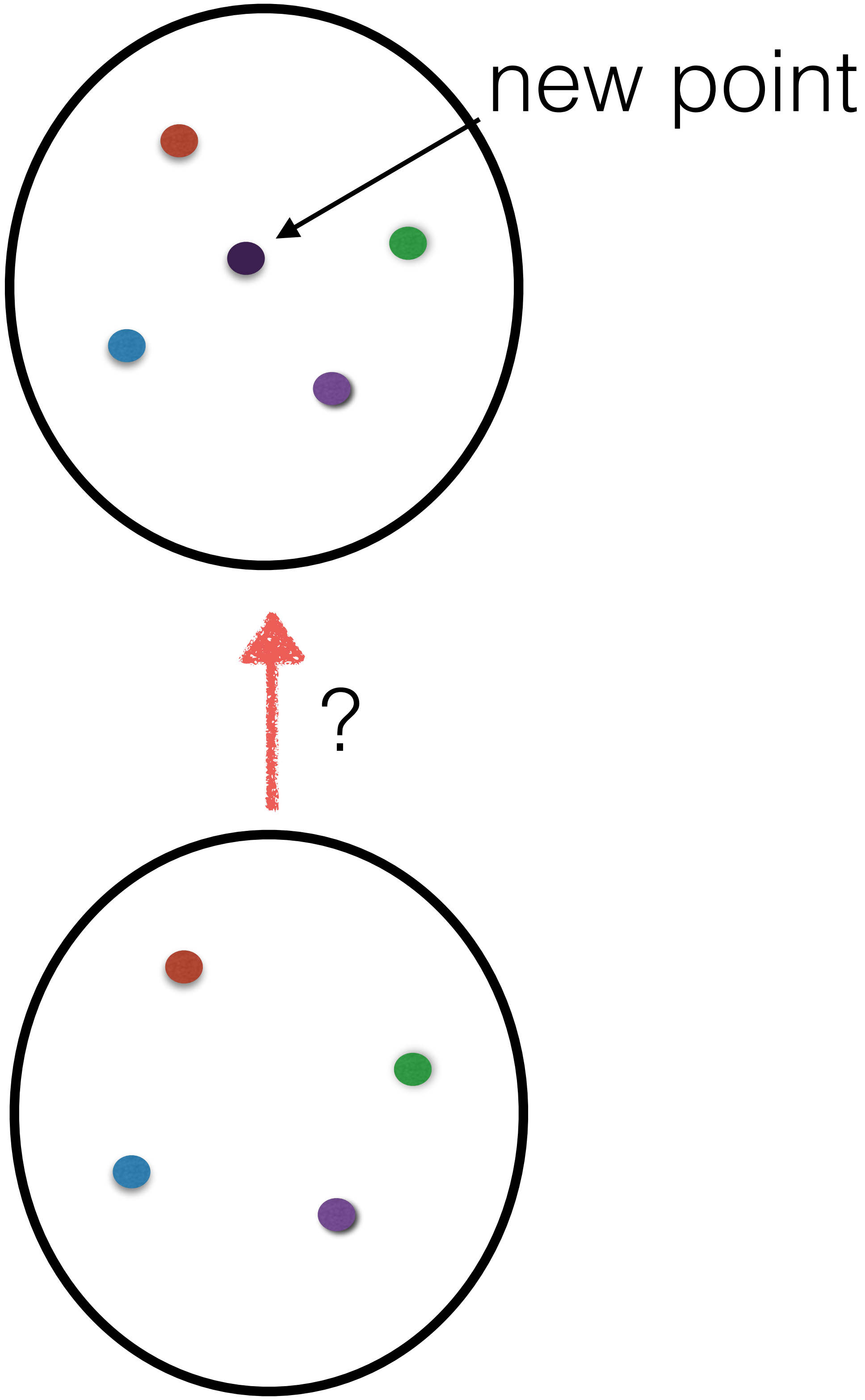

Example 1

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$$\mathrm{Conf}_4(S^2) \xrightarrow{Forget} \mathrm{Conf}_3(S^2)$$

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$$\begin{tikzcd}
\mathrm{Conf}_4(S^2) \arrow[r, "Forget"] & \mathrm{Conf}_3(S^2) \arrow[l, bend left]
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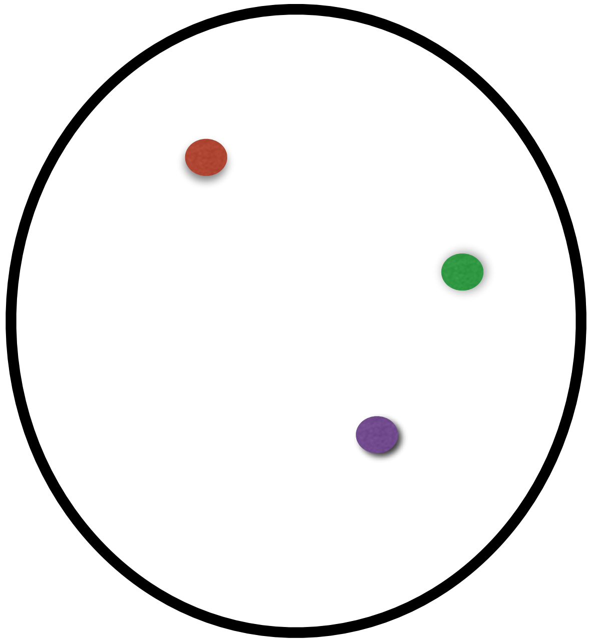

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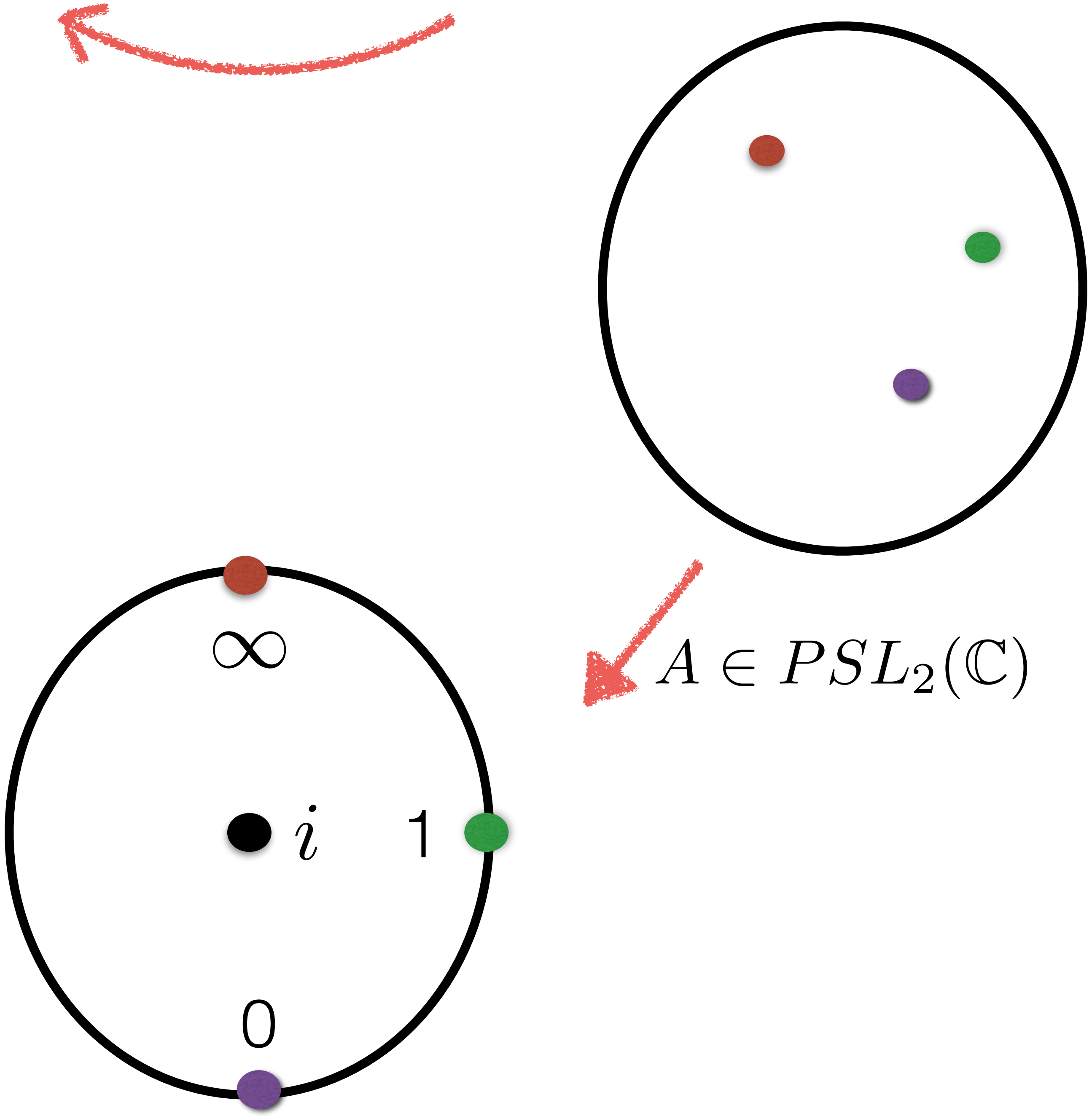

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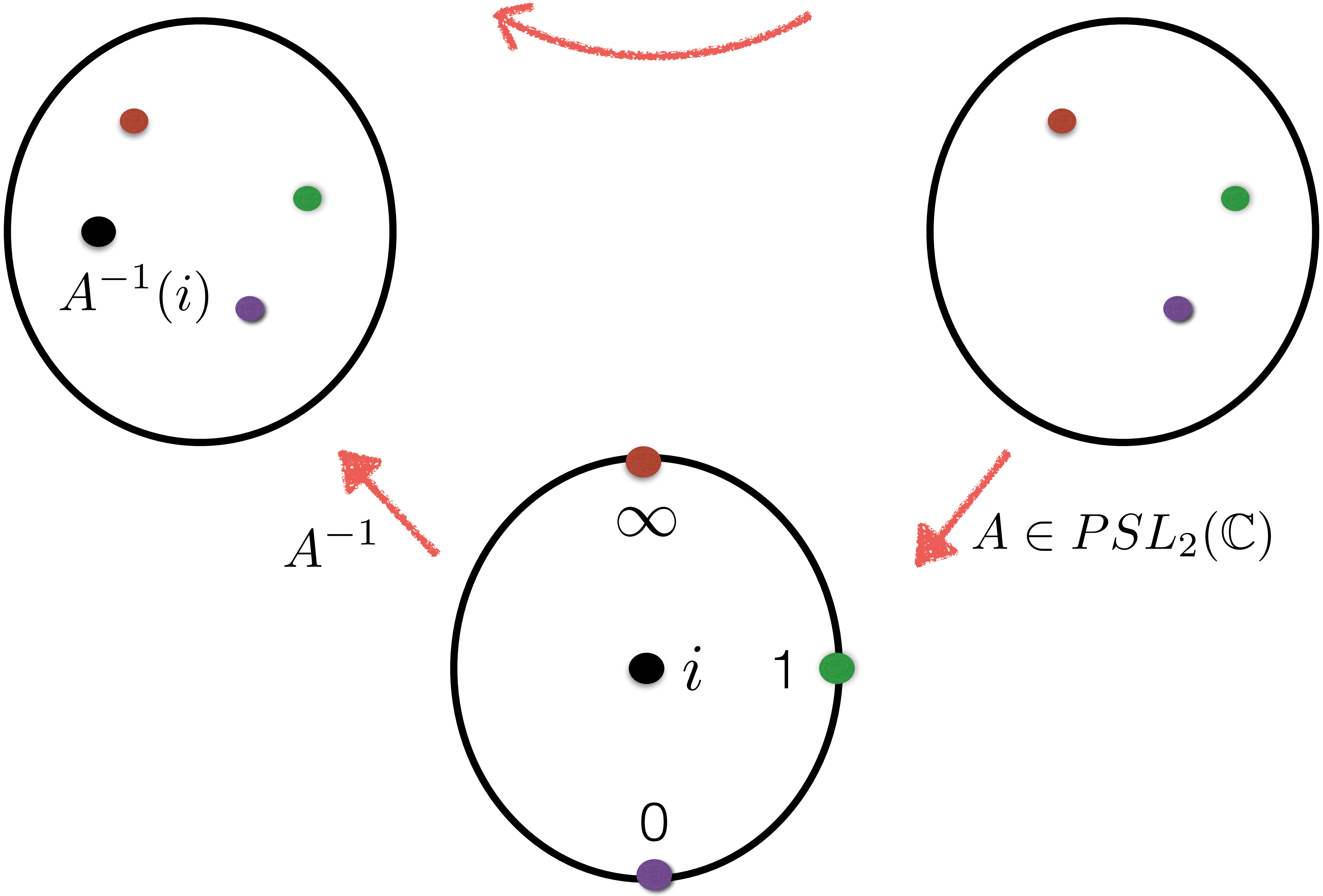

Example 2: $\mathbf{R^2}$

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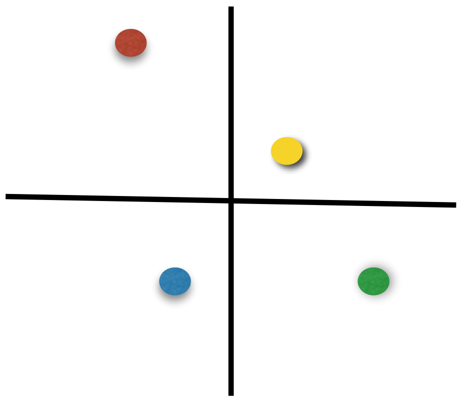
Add far away



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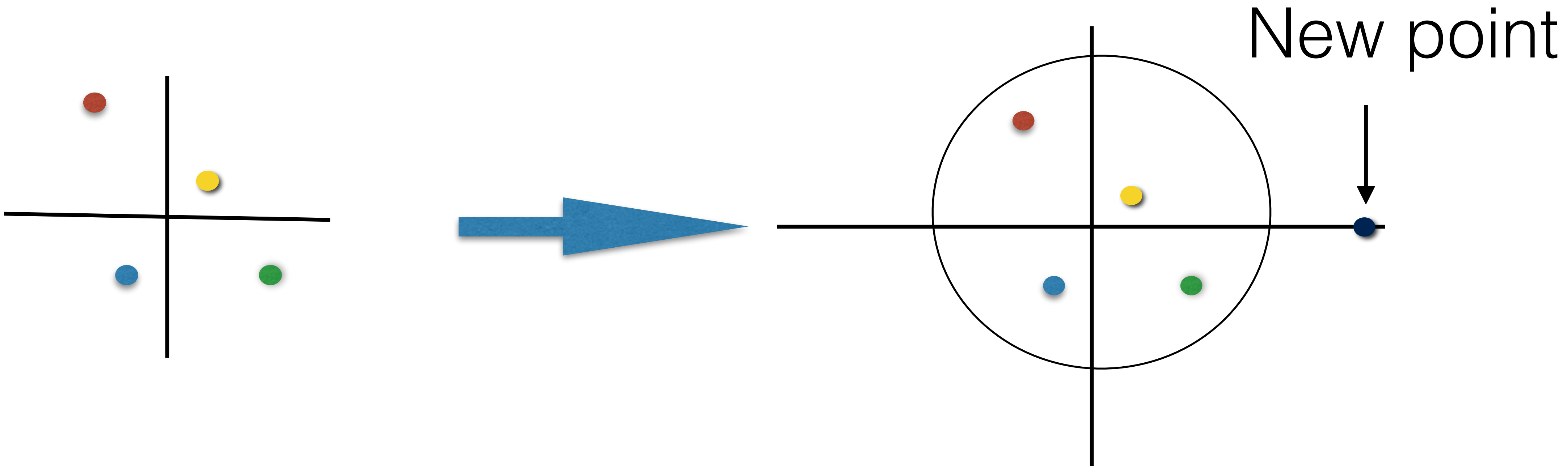

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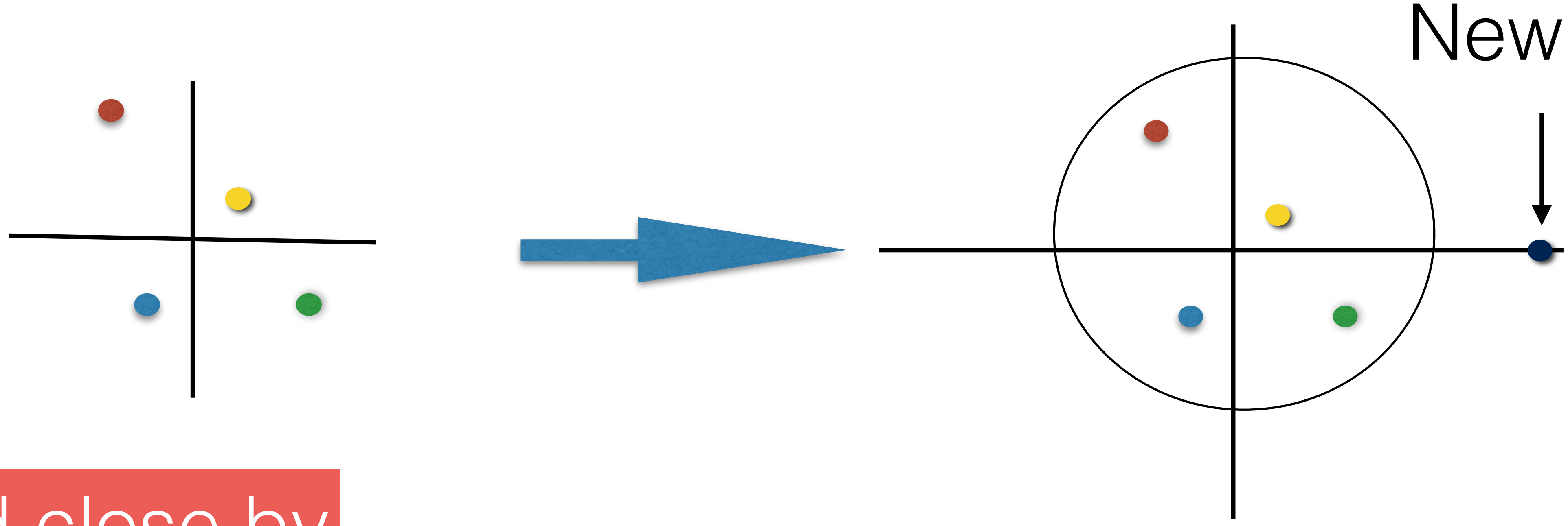
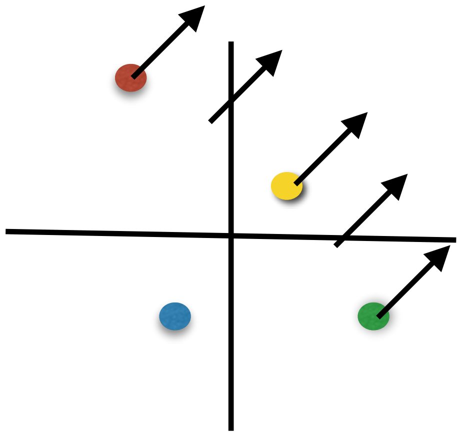


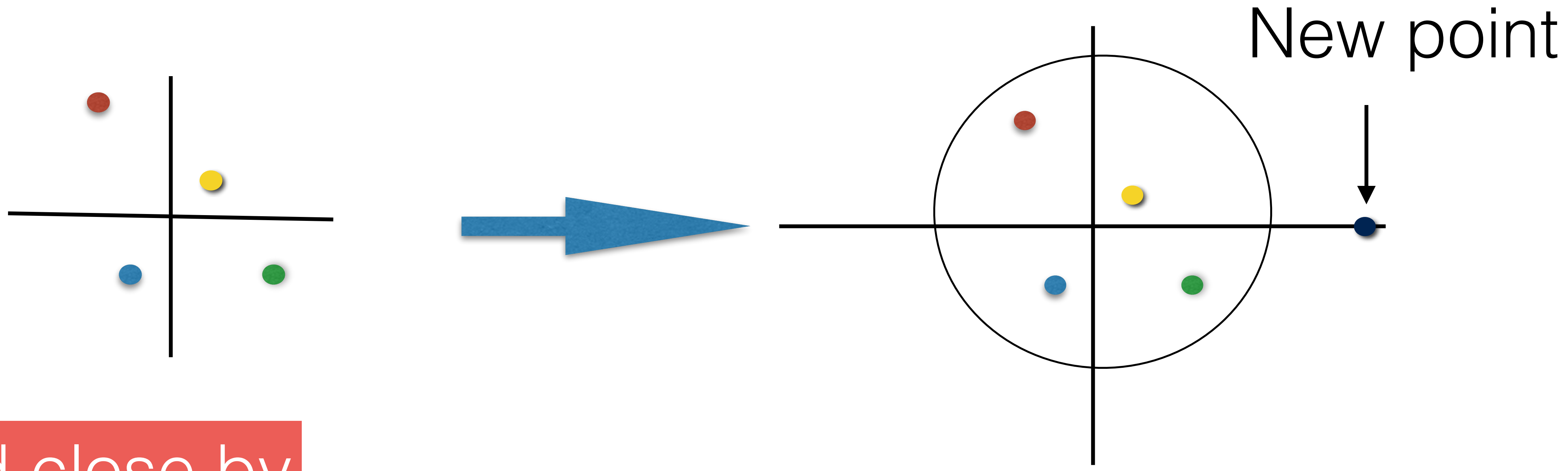
Add close by



vector field

Example 2: $\mathbf{R}^2$

Add far away



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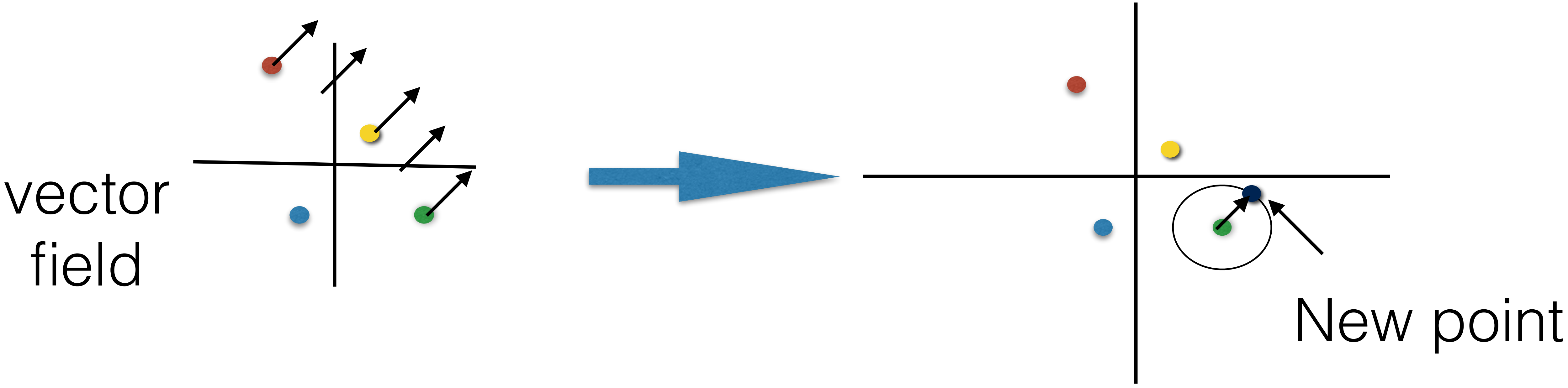

Theorem (C-)

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$$\begin{tikzcd} \mathrm{Conf}_{n+1}(M) \arrow[r, "forget"] & \mathrm{Conf}_n(M)? \arrow[l, bend left, "\exists"] \end{tikzcd}$$

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“Add close by”

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No for n=2

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Only "Add far away"
"Add close by"

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For n>4,
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For n>4,
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3) $g > 1, M = S_g$

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$3)\ g > 1, M = S_g$

No for n>1

Theorem (C-) When does

$$\begin{tikzcd} \mathrm{Conf}_{n+1}(M) \arrow[r, "forget"] & \mathrm{Conf}_n(M)? \arrow[l, bend left, "\exists"] \end{tikzcd}$$

1) $n > 3, M = \mathbb{R}^2$ → Only "Add far away" "Add close by"

2) $M = S^2$

No for n=2 | **Yes for n>2**

For n>4,
only "Add close by"

3) $g > 1, M = S_g$

No for n>1 | **Yes for n=1**

Methods used

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Thurston’s normal form

Thurston, Birman-Lubotzky-McCarthy, Ivanov and others

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Obstructions in $H^*(\mathrm{Conf}_n(M);\mathbf{Q})$

Problem: Other M?

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$$\begin{tikzcd}
\mathrm{Conf}_3(\mathbb{C}P^2) \arrow[r, "forget"] & \mathrm{Conf}_2(\mathbb{C}P^2) \arrow[l, dashed, bend right, "?"']
\end{tikzcd}$$

$$\begin{tikzcd}
\mathrm{Conf}_3(\mathbb{R}P^2) \arrow[r, "forget"] & \mathrm{Conf}_2(\mathbb{R}P^2) \arrow[l, dashed, bend right, "?"']
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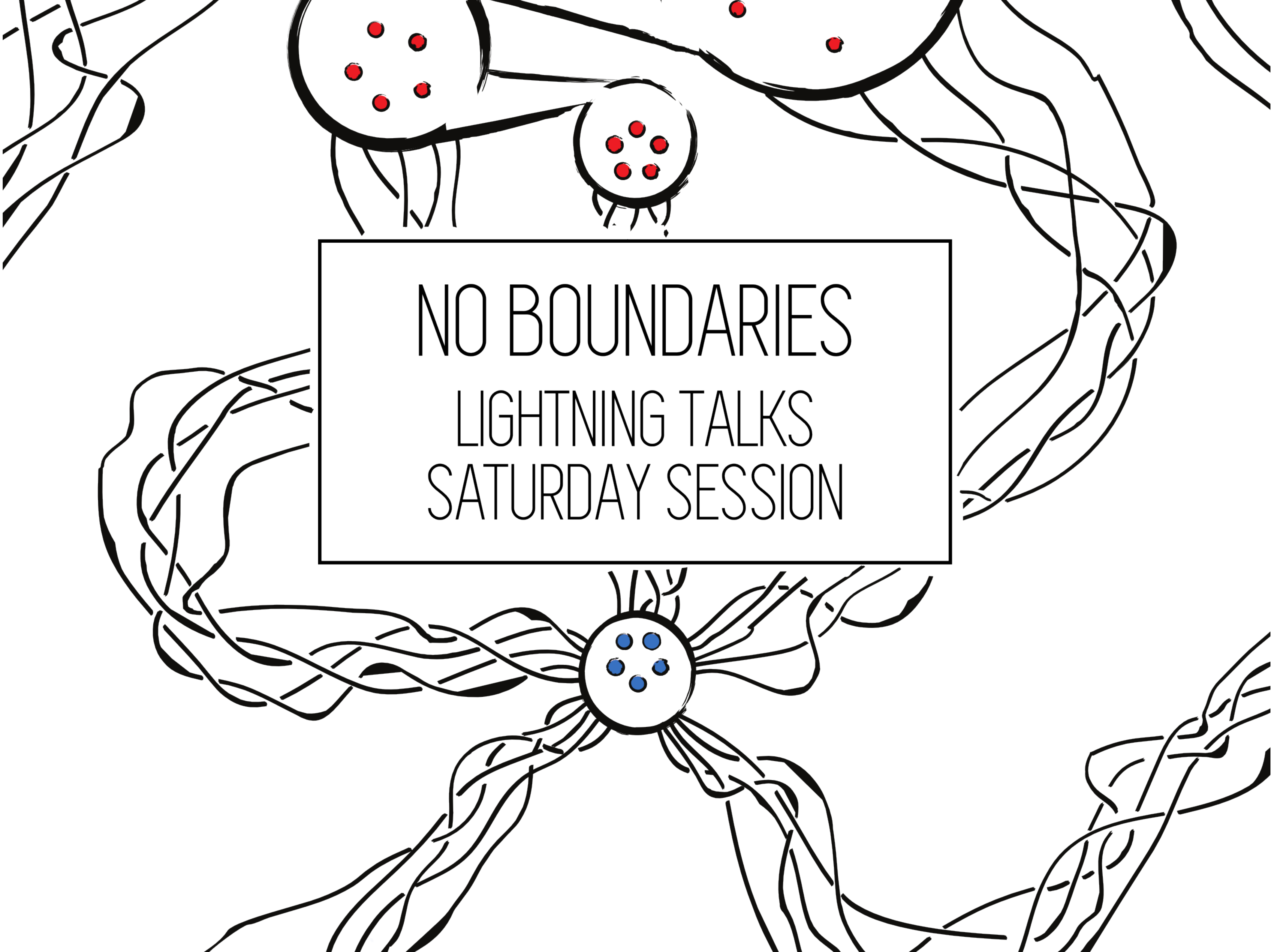
Do we have other examples like these? Exotic sections?

Reference:

Pre-print: Section problems for configuration spaces of surfaces

https://arxiv.org/abs/1708.07921

Thanks!!



NO BOUNDARIES

LIGHTNING TALKS
SATURDAY SESSION

Mapping class groups:
Bigger. Better? Commensurably rigid!

Spencer Dowdall
(with Juliette Bavard & Kasra Rafi)

Vanderbilt University
math.vanderbilt.edu/dowdalsd/

No boundaries conference
October 28, 2017

Mapping class groups

S – oriented surface without boundary

$$\mathrm{Mod}(S) = \mathrm{Homeo}(S)/\text{isotopy}$$

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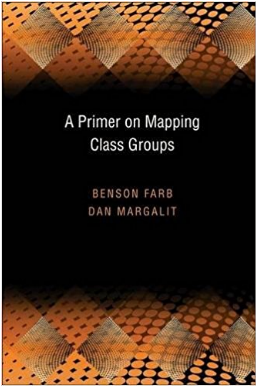

A Primer on Mapping
Class Groups

BENSON FARB
DAN MARGALIT

Mapping class groups

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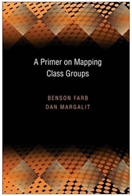

A Primer on Mapping Class Groups

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Go Bigger!

Big mapping class groups: Mod(infinite-type surface)

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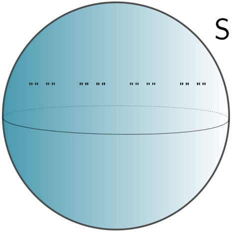
Big mapping class groups: $\mathrm{Mod}(\text{infinite-type surface})$



Sphere minus Cantor set

Go Bigger!

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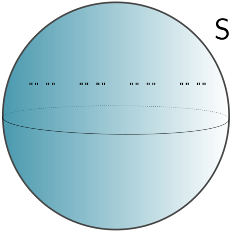
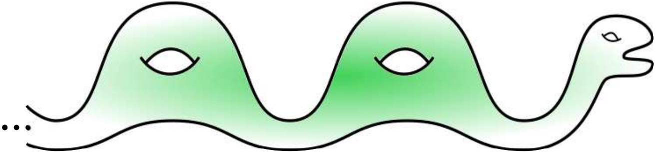


Sphere minus Cantor set

Loch Ness Monster surface



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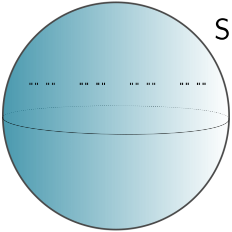

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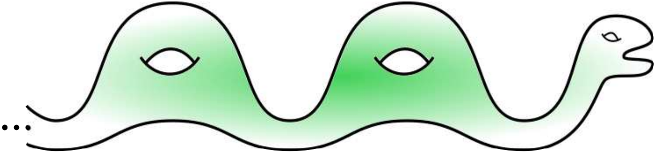

Loch Ness Monster surface

Here be dragons!

- uncountable
- $\mathrm{Mod}(S)$ inherits nondiscrete topology from $\mathrm{Homeo}(S)$

Recognition and Rigidity

Question: Do big mapping class groups distinguish surfaces?

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S_1 and S_2 infinite-type surfaces. Any isomorphism $G_1 \to G_2$ between finite index subgroups G_i of $\mathrm{Mod}(S_i)$ is induced by a homeomorphism $S_1 \to S_2$.

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Abstract Commensurator

$\mathrm{Comm}(G) =$ group of isomorphisms between finite-index subgroups

- E.g: $\mathrm{Aut}(\mathbb{Z}) \cong \mathbb{Z}/2\mathbb{Z}$, but $\mathrm{Comm}(\mathbb{Z}) = \mathbb{Q}^*$

Ingredients

1) Algebraic identification of Dehn twists:

Key Lemma (Bavard–D.–Rafi)

$f \in \mathrm{Mod}(S)$ *has finite-support* $\iff$ f*'s conjugacy class is countable.*

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1) Algebraic identification of Dehn twists:

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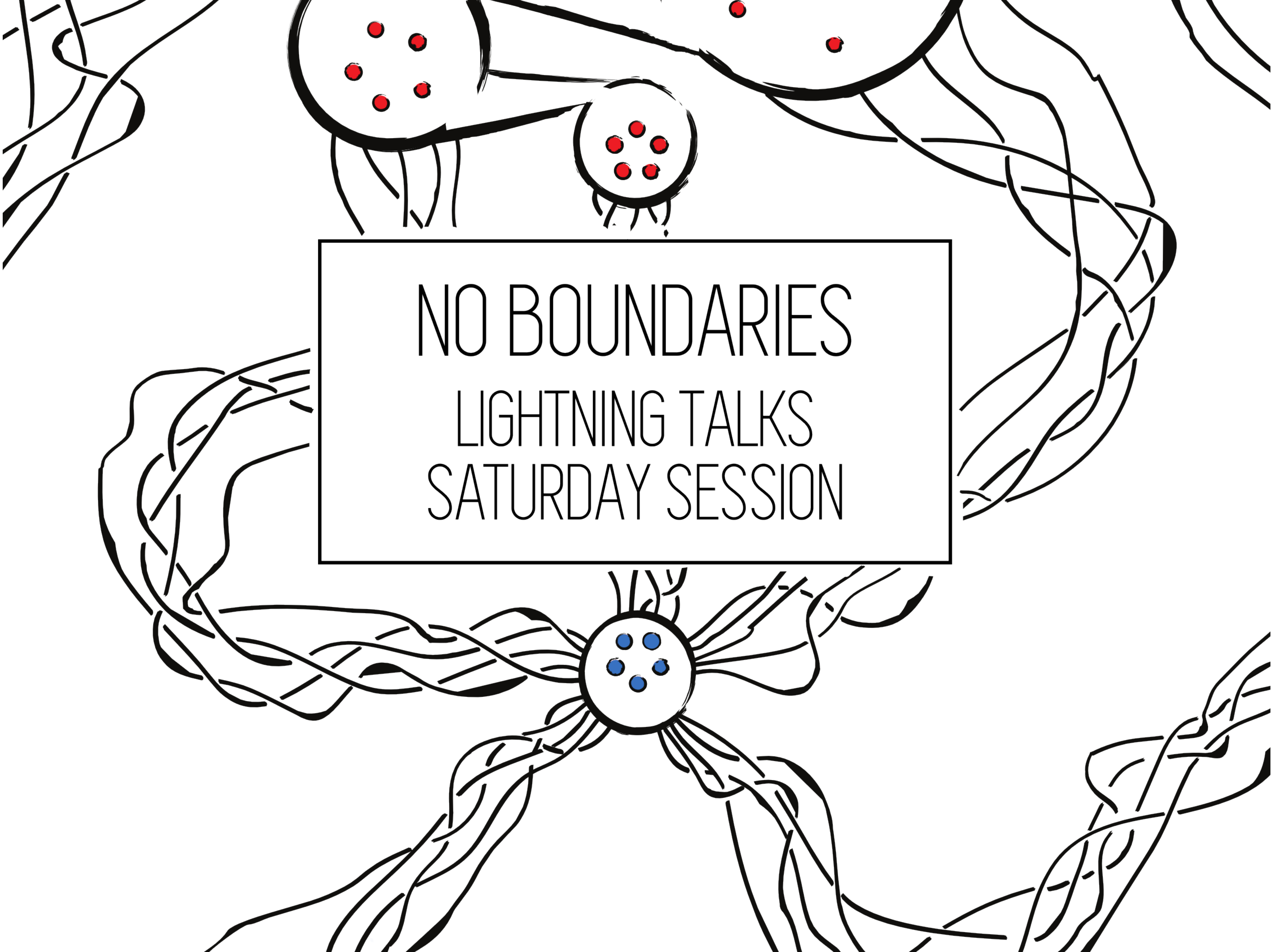
$f \in \mathrm{Mod}(S)$ *has finite-support* $\iff$ f*'s conjugacy class is countable.*

2) Rigidity of curve complexes $\mathcal{C}(S)$:

Theorem (Hernandez–Morales–Valdez; Bavard–D.–Rafi)

S_1 *and* S_2 *infinite-type surfaces. Any automorphism* $\mathcal{C}(S_1) \to \mathcal{C}(S_2)$ *of their curve complexes is induced by a homeomorphism* $S_1 \to S_2$.



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