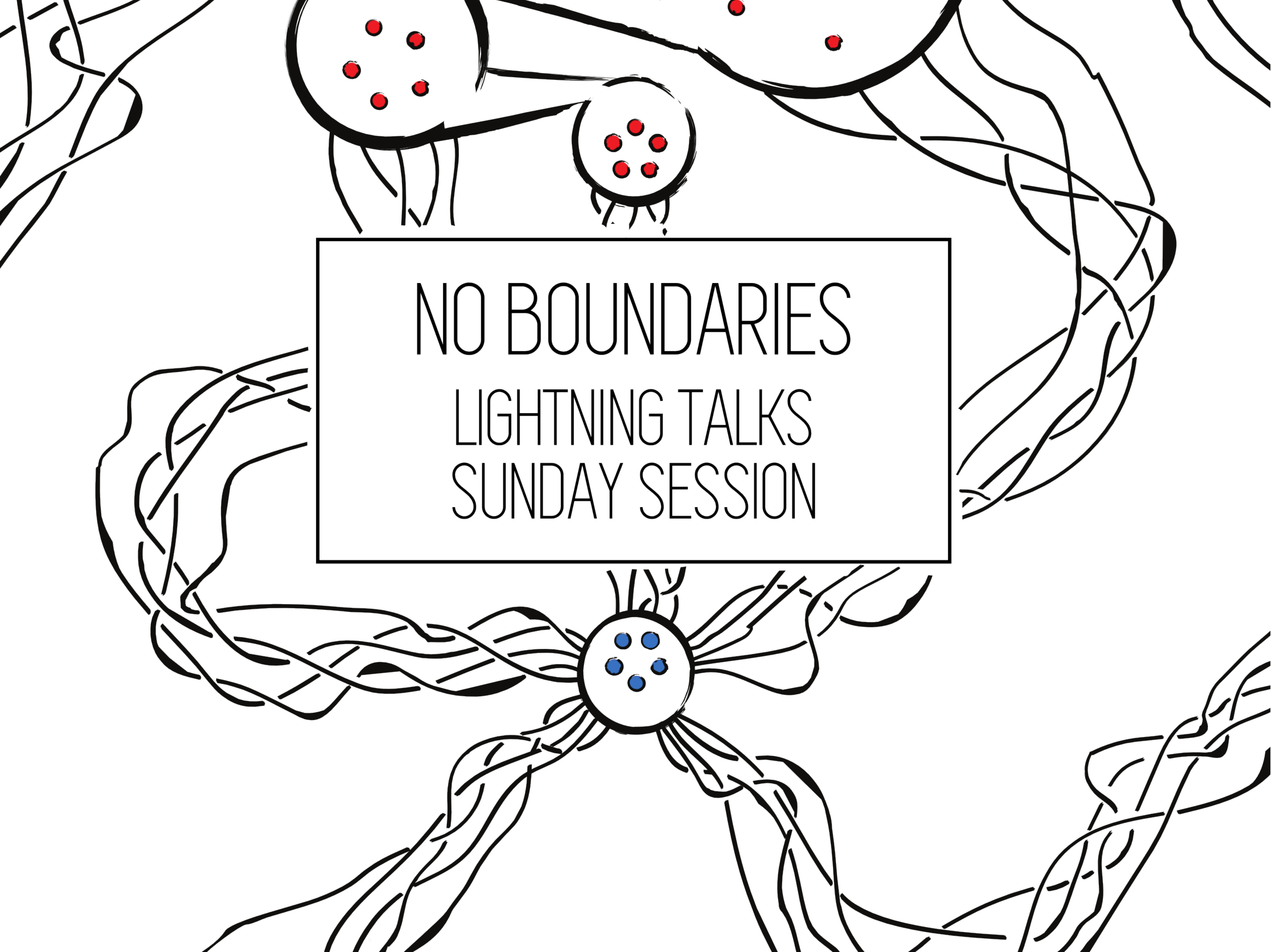


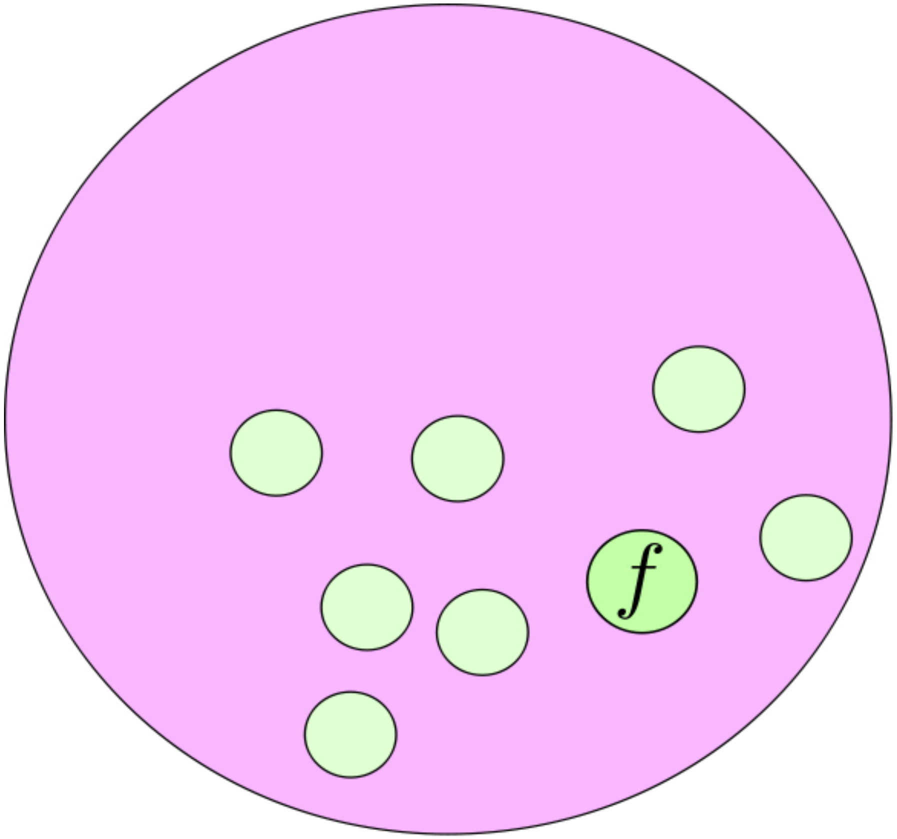
NO BOUNDARIES

LIGHTNING TALKS
SUNDAY SESSION

Normal generators for mapping class groups are abundant.

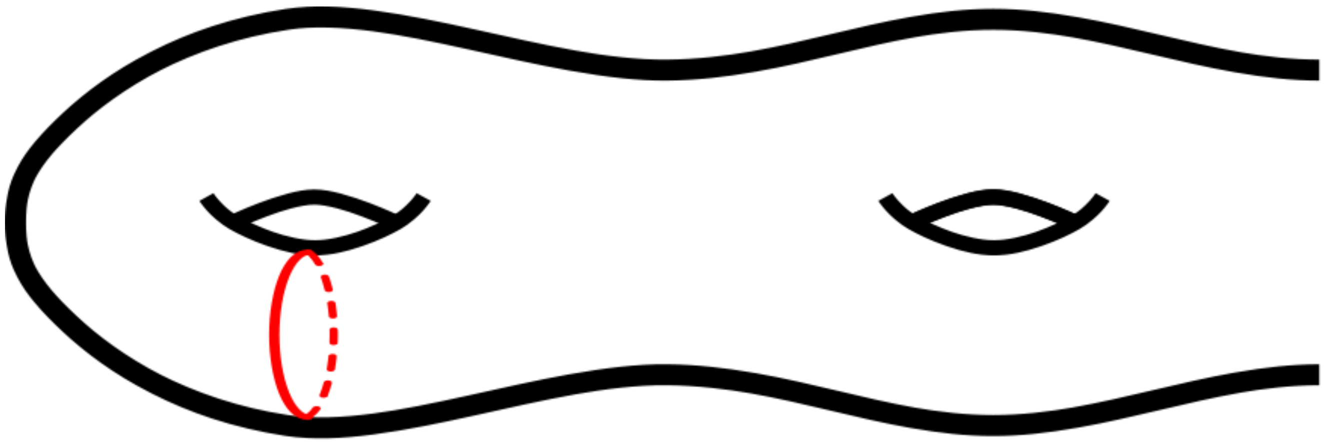
Justin Lanier
Georgia Tech
(joint with Dan Margalit)

$\mathrm{Mod}(S_g)$



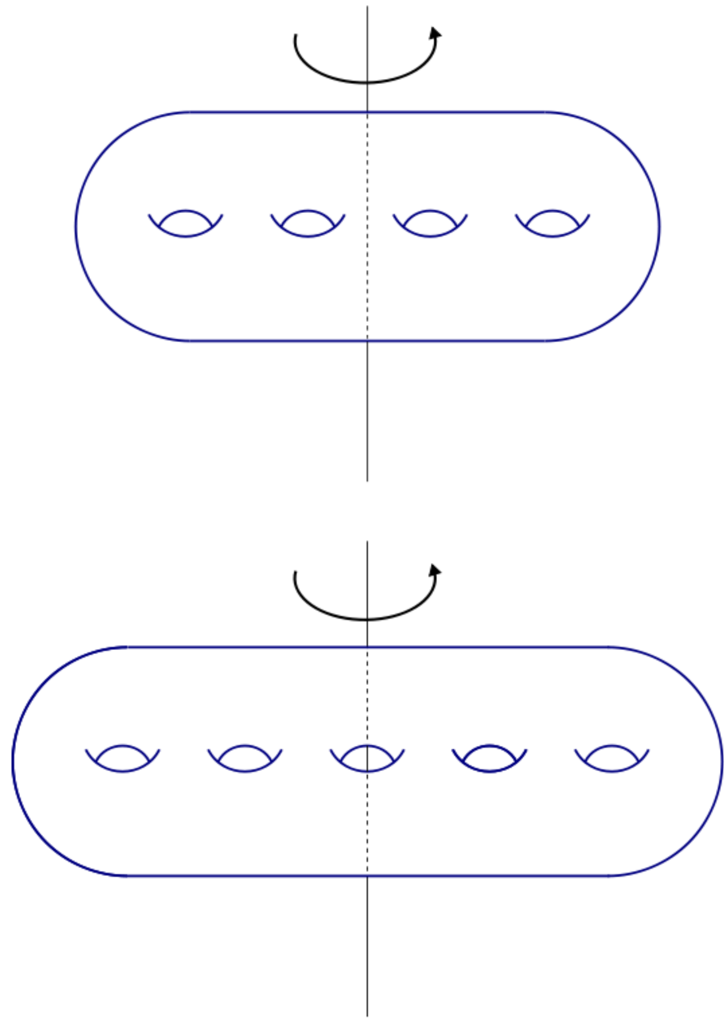
normal generator:

$\langle$ conjugates of f $\rangle = \mathrm{Mod}(S_g)$



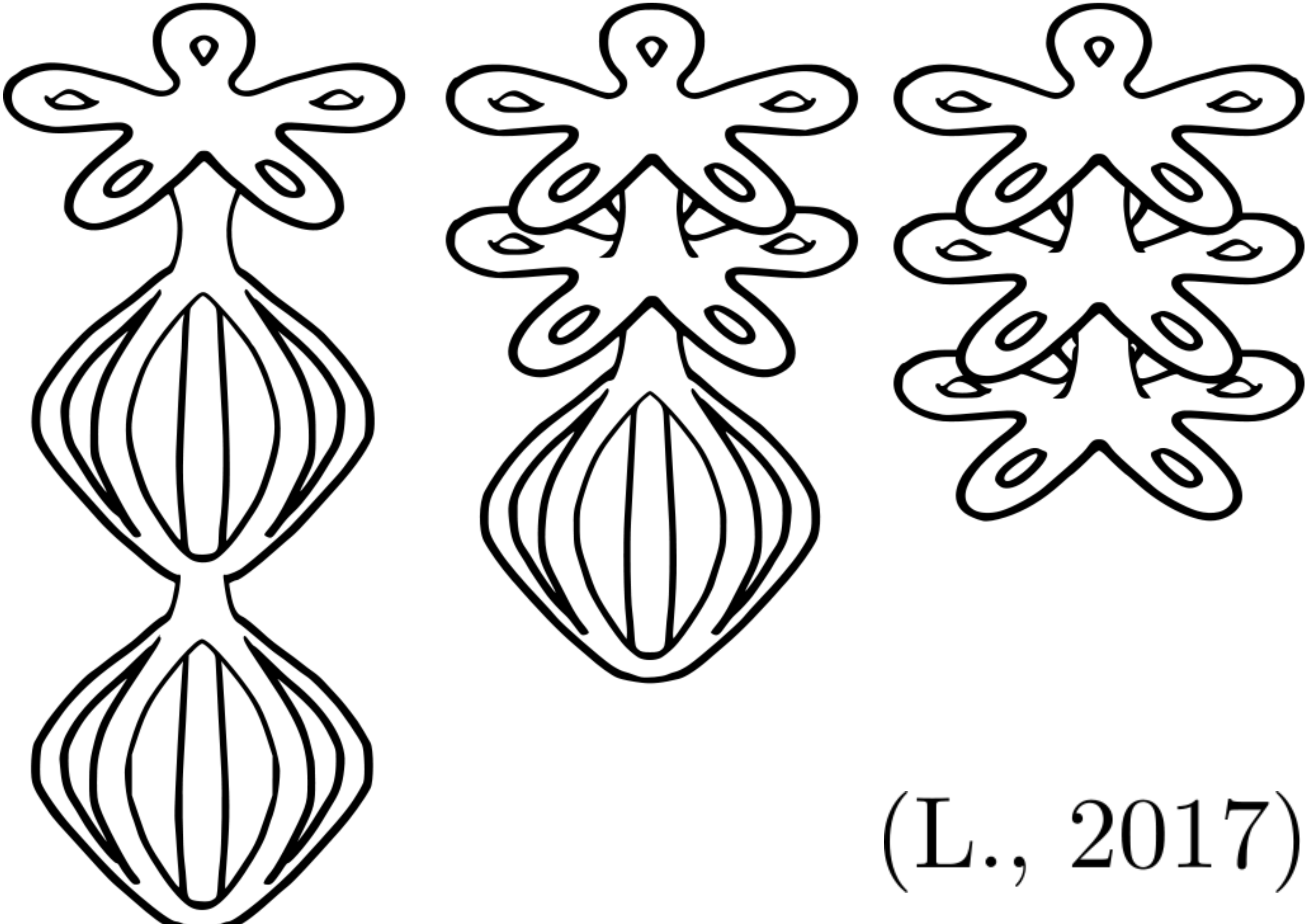
Dehn twist

order 2, $g \geq 3$

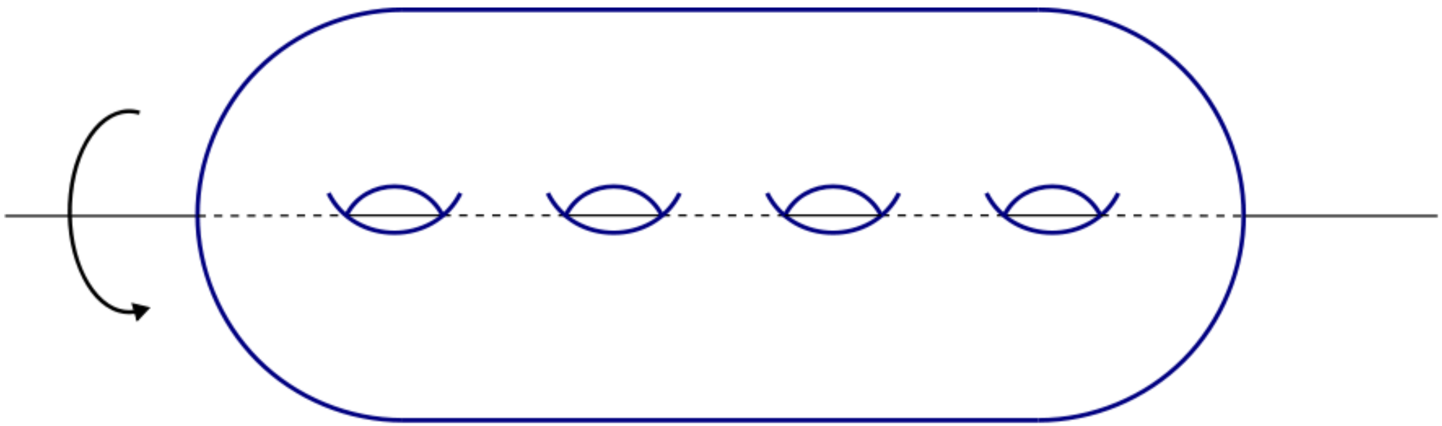


(McCarthy-Papadopoulos, 1987)

order ≥ 3, g large



(L., 2017)

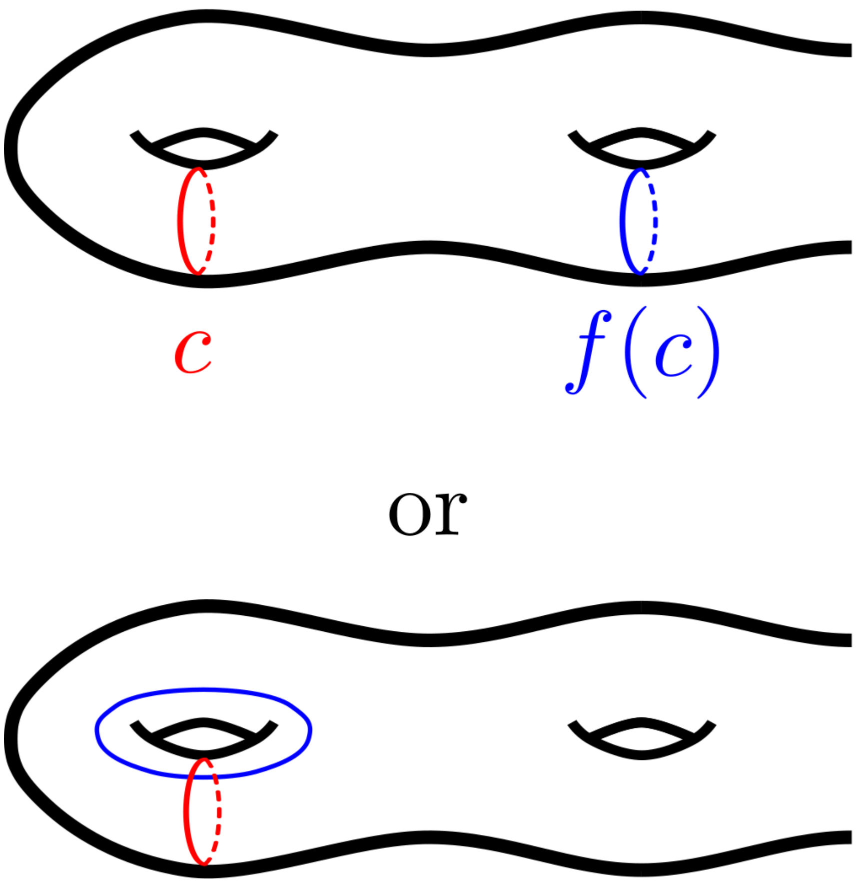


hyperelliptic involution

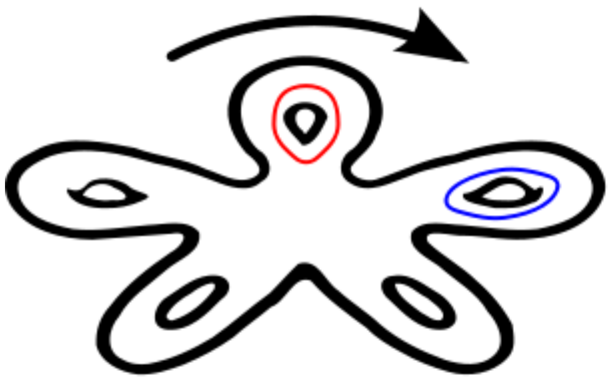
Theorem (L.-Margalit, 2017)

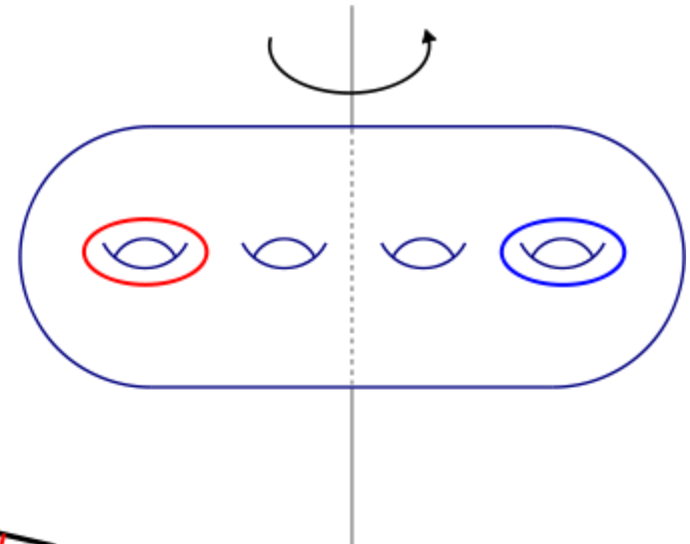
For $g \geq 3$, every periodic mapping class that is not a hyperelliptic involution normally generates $\mathrm{Mod}(S_g)$.

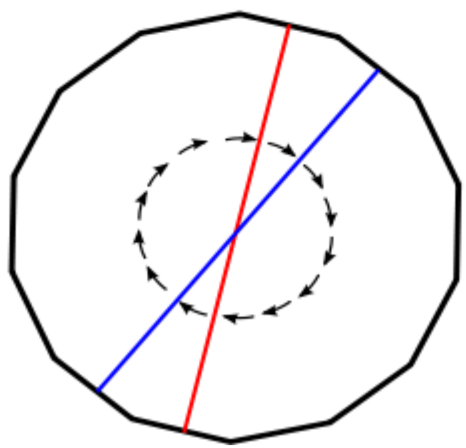
Well-suited curve criteria



$\Longrightarrow$ normal generator







Question (Long, 1986)

Can the normal closure of a (pseudo-)Anosov map ever be all of $\mathrm{Mod}(S_g)$?

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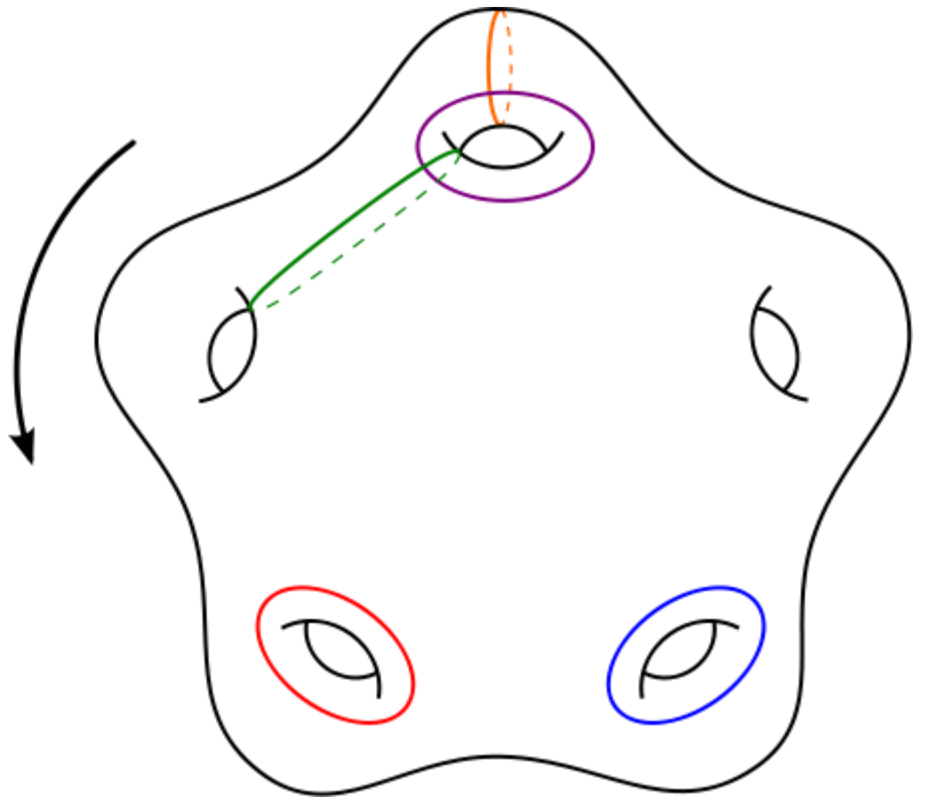
Can the normal closure of a (pseudo-)Anosov map ever be all of $\mathrm{Mod}(S_g)$?

Answer: **Yes!**



(Penner, 1988)

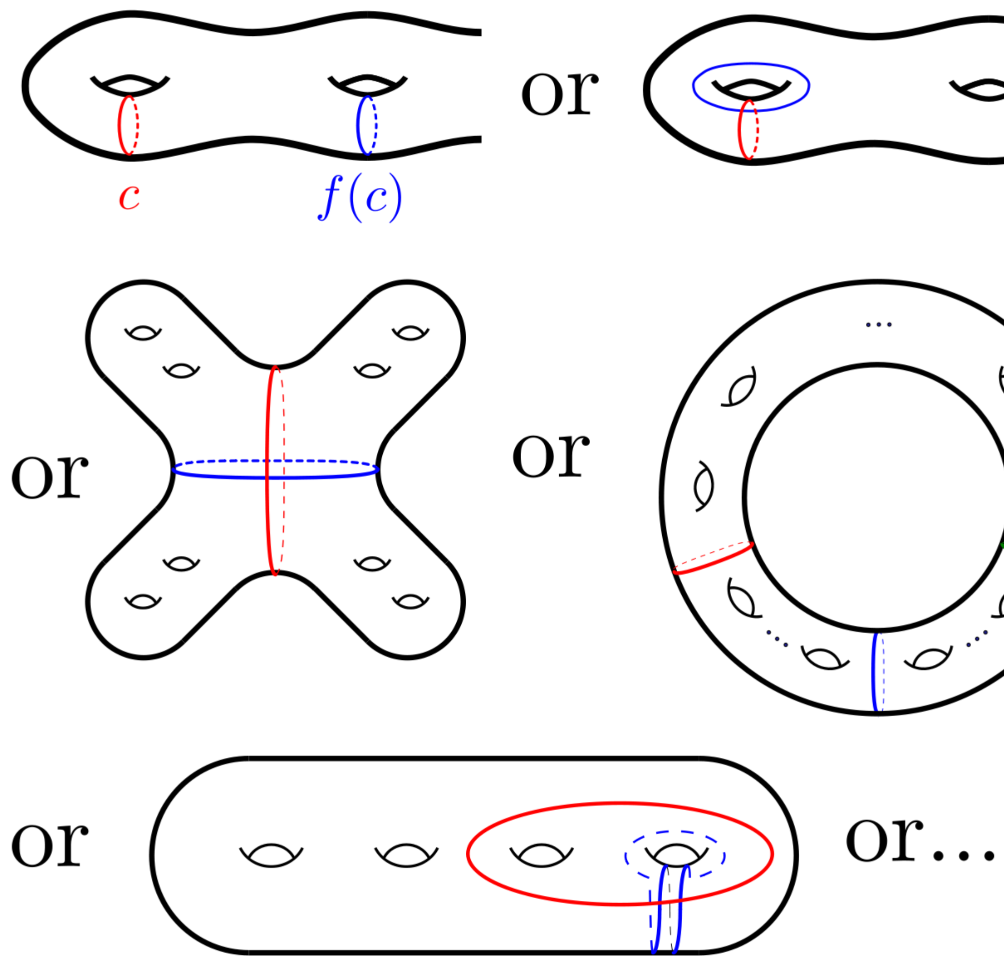
Theorem (L.-Margalit, 2017)

For $g \geq 3$, every pseudo-Anosov element with stretch factor less than 1.1 normally generates $\mathrm{Mod}(S_g)$.

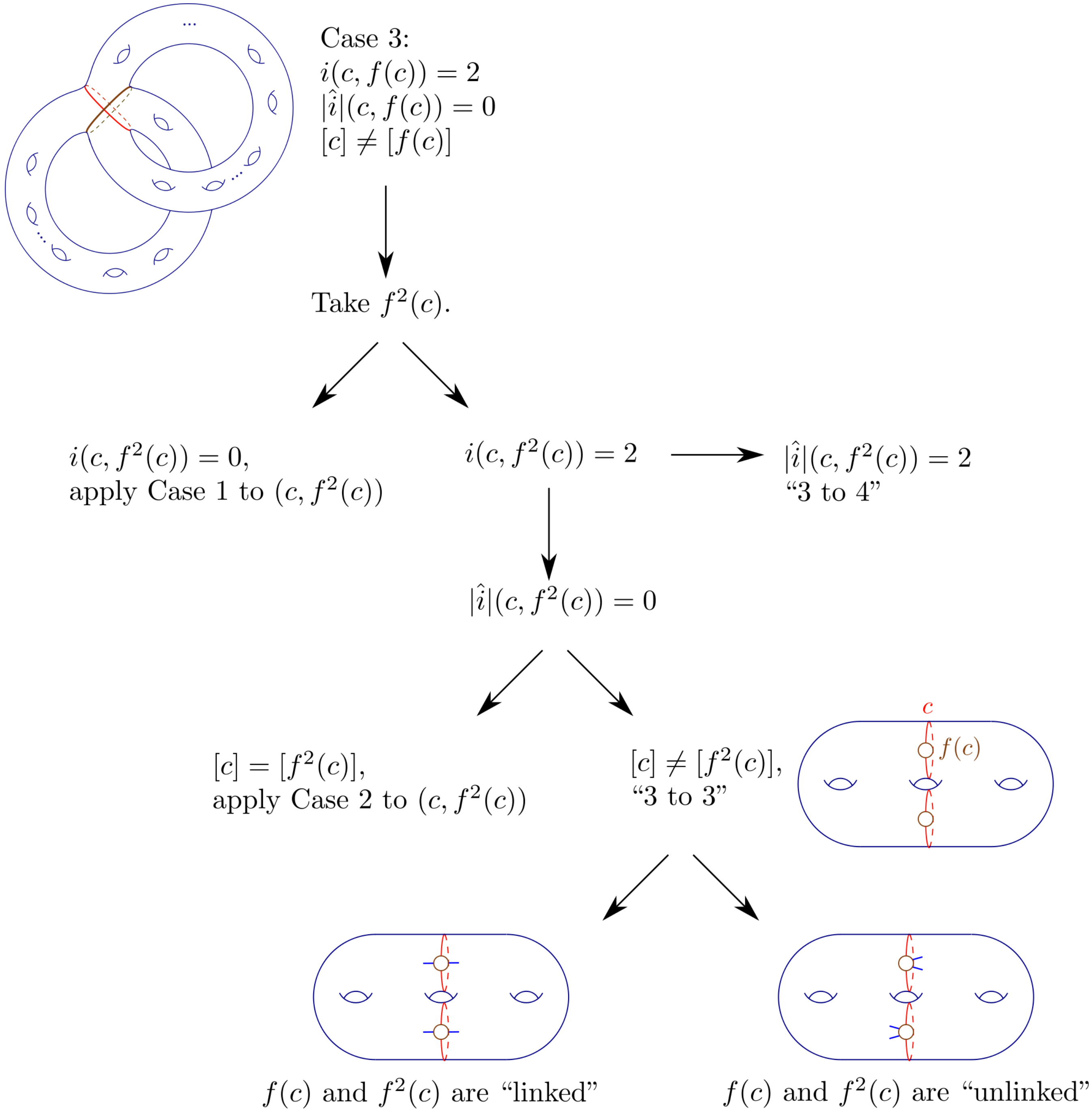
f with stretch factor less than $3/2$ $\implies$ short curve c with $i(c, f(c)) \leq 2$

(Farb-Leininger-Margalit, 2011)

Well-suited curve criteria 2.0!



normal generator



Case 3:
$i(c, f(c)) = 2$
$|\hat{i}|(c, f(c)) = 0$
$[c] \neq [f(c)]$
Take $f^2(c)$.
$i(c, f^2(c)) = 0$, apply Case 1 to $(c, f^2(c))$
$i(c, f^2(c)) = 2$
$|\hat{i}|(c, f^2(c)) = 2$ "3 to 4"
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$[c] \neq [f^2(c)]$, "3 to 3"
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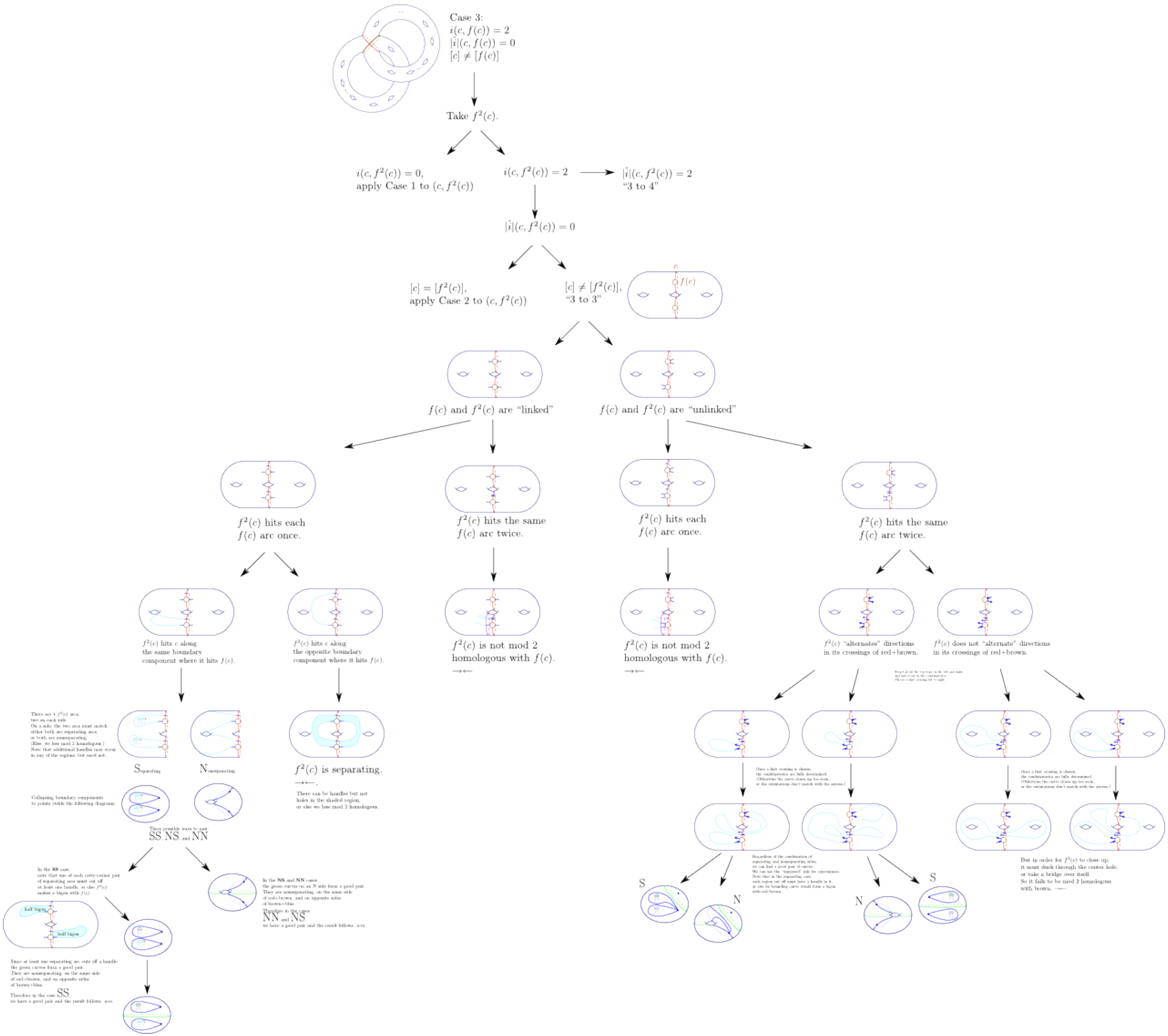

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$f^2(c)$ is not mod 2
homologous with $f(c)$.
$\Rightarrow\Leftarrow$
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$\Rightarrow\Leftarrow$
$f^2(c)$ "alternates" directions
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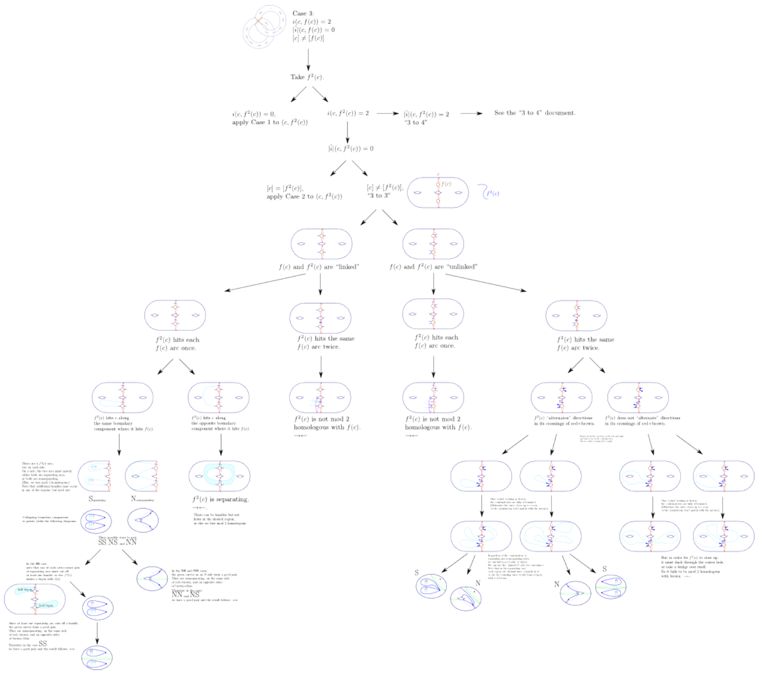
$f^2(c)$ does not "alternate" directions
in its crossings of red+brown.
$S_{\text{separating}}$
$N_{\text{nonseparating}}$
$f^2(c)$ is separating.
$\Rightarrow\Leftarrow$.
There can be handles but not
holes in the shaded region,
or else we lose mod 2 homologous.
Three possible ways to pair:
SS, NS, and NN
half bigon
half bigon
S
N
N
S
But in order for $f^2(c)$ to close up,
it must duck through the center hole,
or take a bridge over itself.
So it fails to be mod 2 homologous
with brown. $\Rightarrow\Leftarrow$.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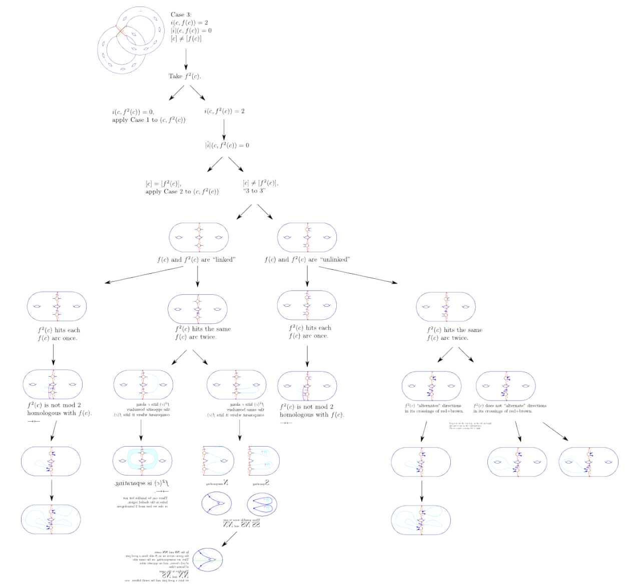
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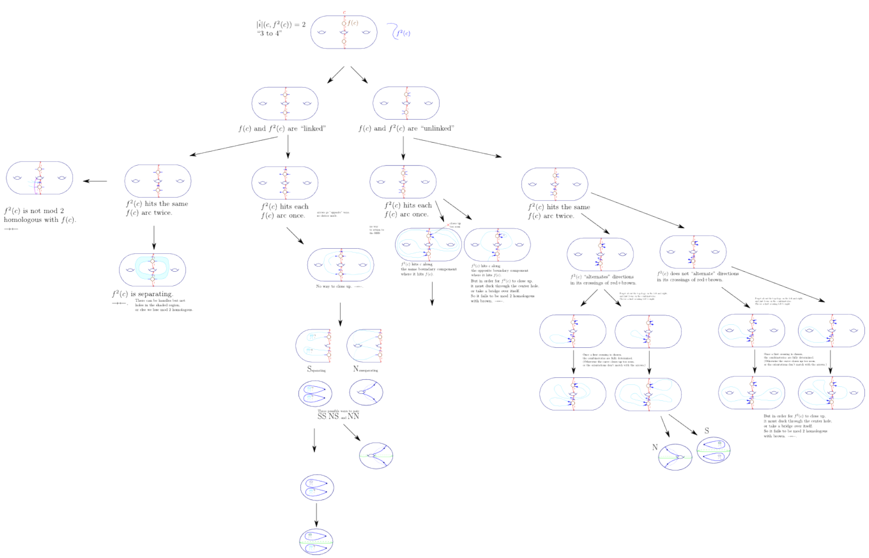
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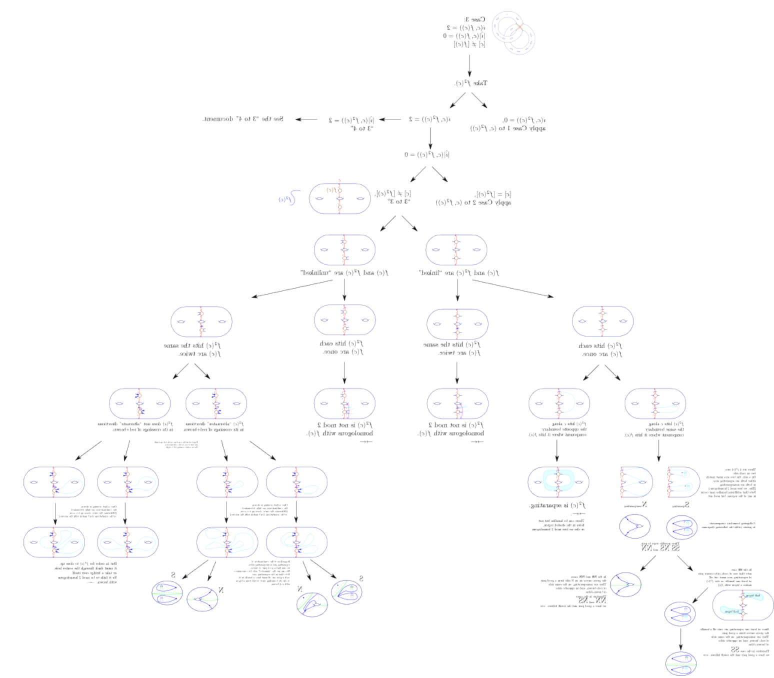

Normal generators
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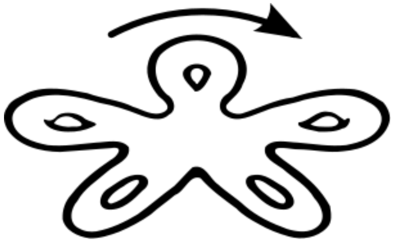

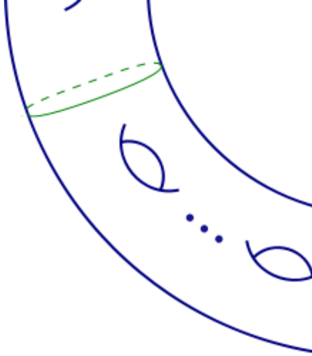

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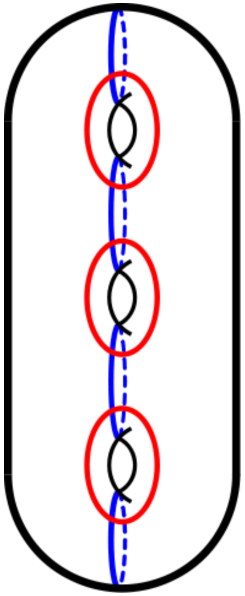

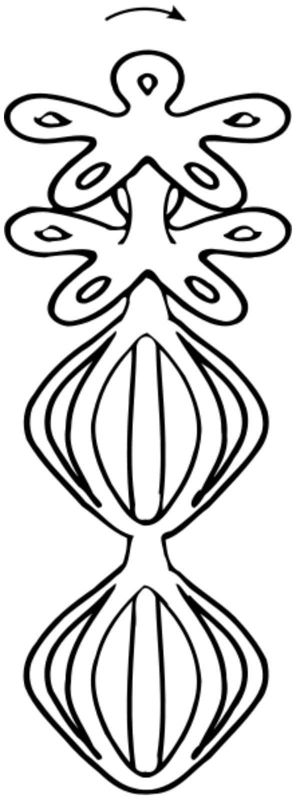

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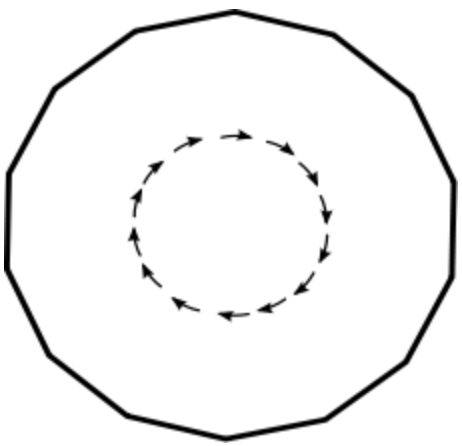
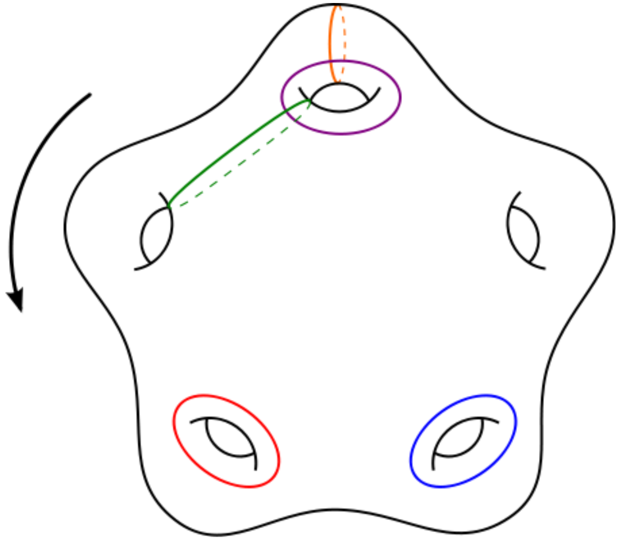
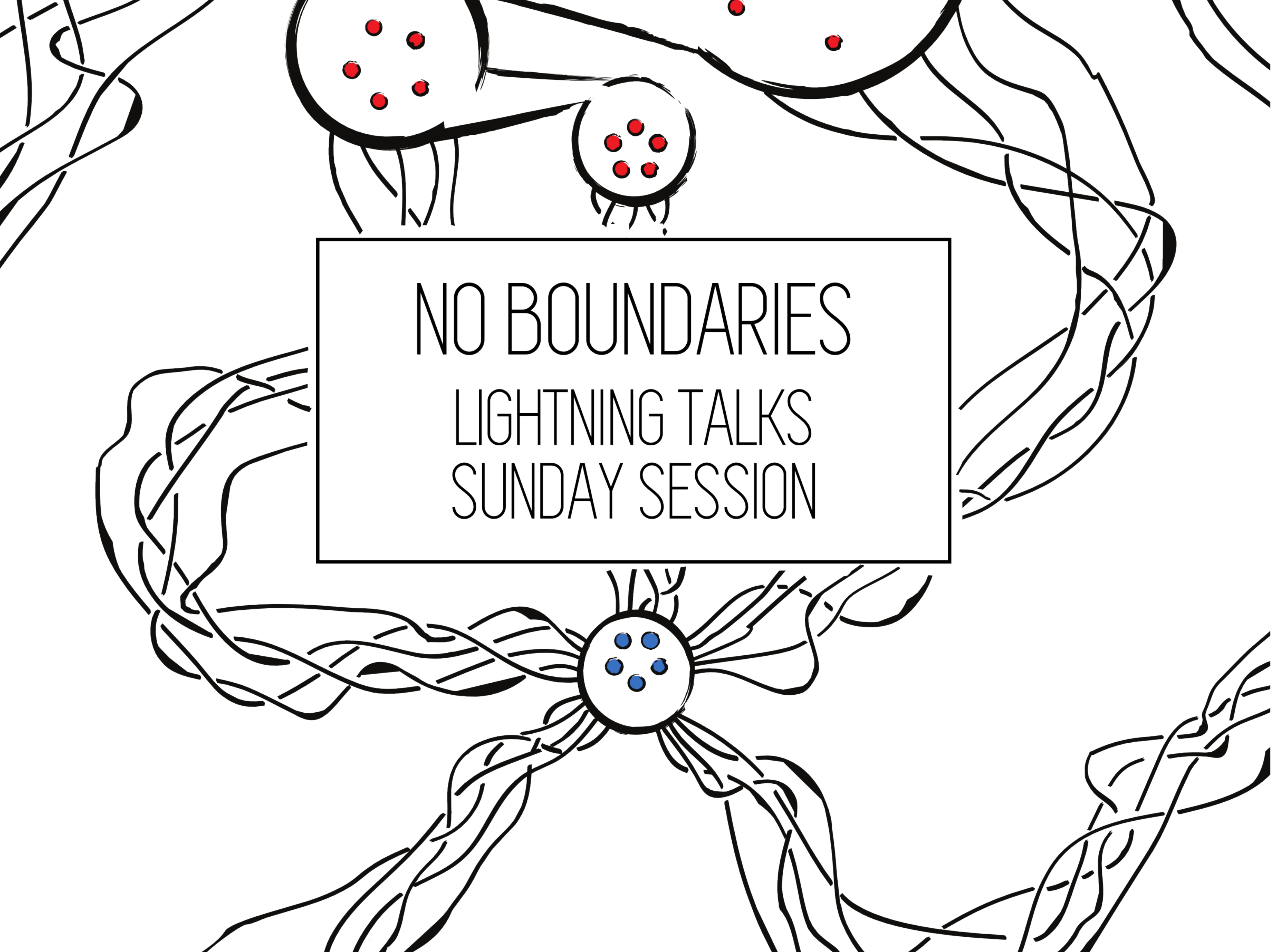




Thanks.


NO BOUNDARIES
LIGHTNING TALKS
SUNDAY SESSION

Hyperbolic structures on groups

Carolyn R. Abbott

University of California, Berkeley

October 28, 2017

Joint work with S. Balasubramanya and D. Osin

Group actions

Goal: Understand groups through their actions on metric spaces.

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Every group admits two actions on metric spaces:

$G \curvearrowright *$ $\qquad$ $G \curvearrowright$ Cayley graph

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- Action encodes all the information about the group

Group actions

Goal: Understand groups through their actions on metric spaces.

Every group admits two actions on metric spaces:

$G \curvearrowright *$

- Action gives no information about the group
- Metric space completely understood

$G \curvearrowright$ Cayley graph

- Action encodes all the information about the group
- Metric space may be extremely complicated

Ordering group actions

We define a partial order on set of isometric actions of G.

Definition

Given isometric actions of a group G on metric spaces R and S, we say

$$G \curvearrowright S \preceq G \curvearrowright R$$

if there is a coarsely G–equivariant Lipschitz map $R \to S$.

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Example:

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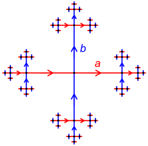

Poset of hyperbolic structures

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Poset of hyperbolic structures

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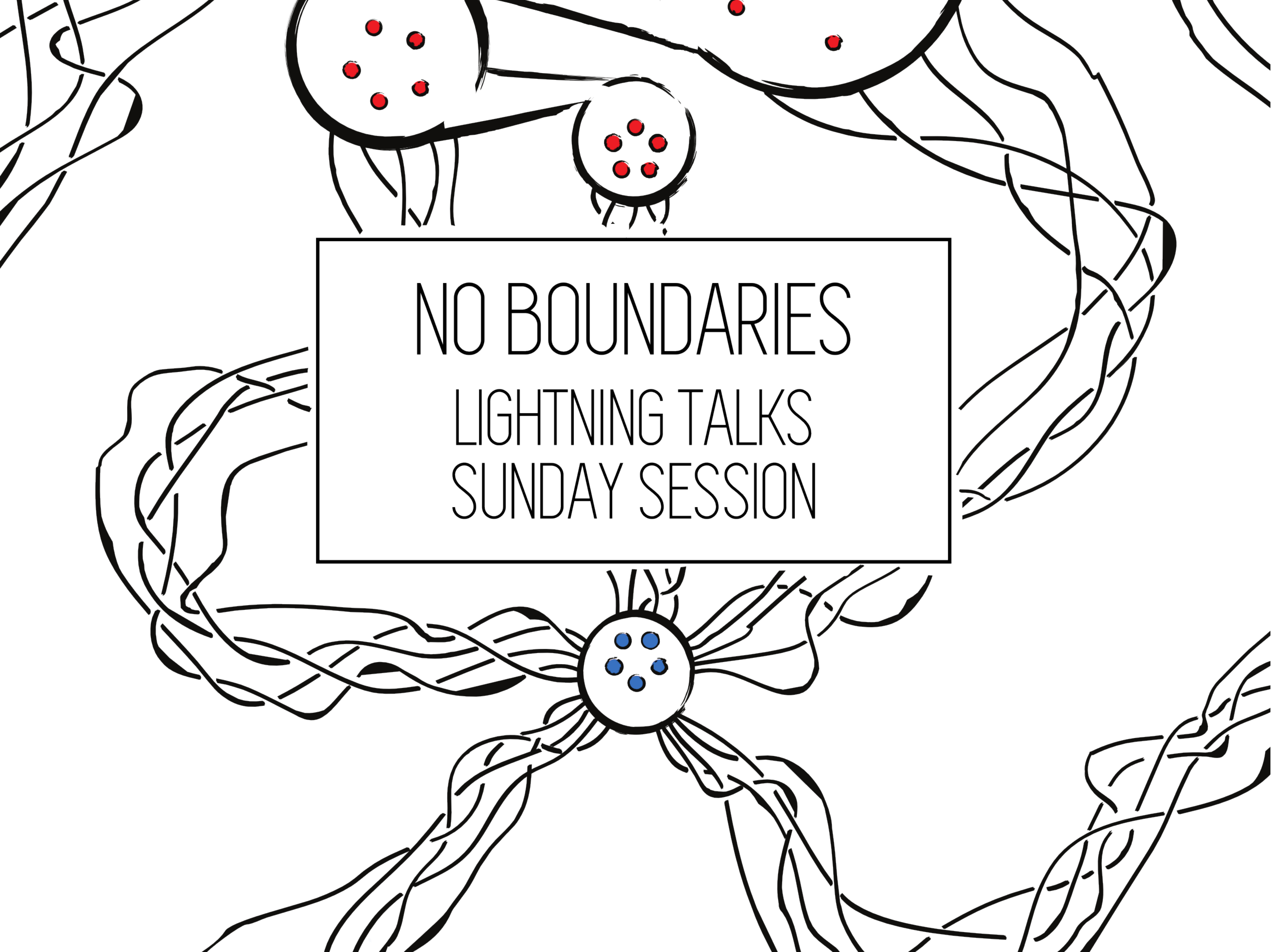
- Is $\mathcal{H}(G)$ a lattice?
- When does $\mathcal{H}(G)$ contain a largest element?


NO BOUNDARIES
LIGHTNING TALKS
SUNDAY SESSION

How many points can be chosen continuously on smooth cubic plane curves?

Weiyan Chen

University of Minnesota, Twin Cities.

No Boundaries: Groups in Algebra, Geometry, and Topology,
A Celebration of the Mathematical Contributions of Benson Farb
University of Chicago
October 29, 2017.

Question

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- A cubic plane curve is given by

$$C_F = \{[x : y : z] \mid F(x, y, z) = 0\} \ \subset \ \mathbb{CP}^2$$

where $F(x, y, z)$ is a homogeneous polynomial of degree 3.

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This leads one to wonder:

- "Is it possible to continuously choose *n* points on any smooth cubic plane curve?"

- "Is $n = 9$ the only possible case?"

- "Is the algebraic construction the only example allowed by topology?"

Reformulating the question

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Define $X := \{F(x, y, z) |$ homogeneous, degree 3, and smooth$\} \subset \mathbb{C}^{10}$.

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where $\mathrm{UConf}^n C_F :=$ the configuration space of n distinct unlabeled points on C_F.

Reformulating the question

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Answers and further questions...

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- What about other values of n (for example, $n = 18$)?
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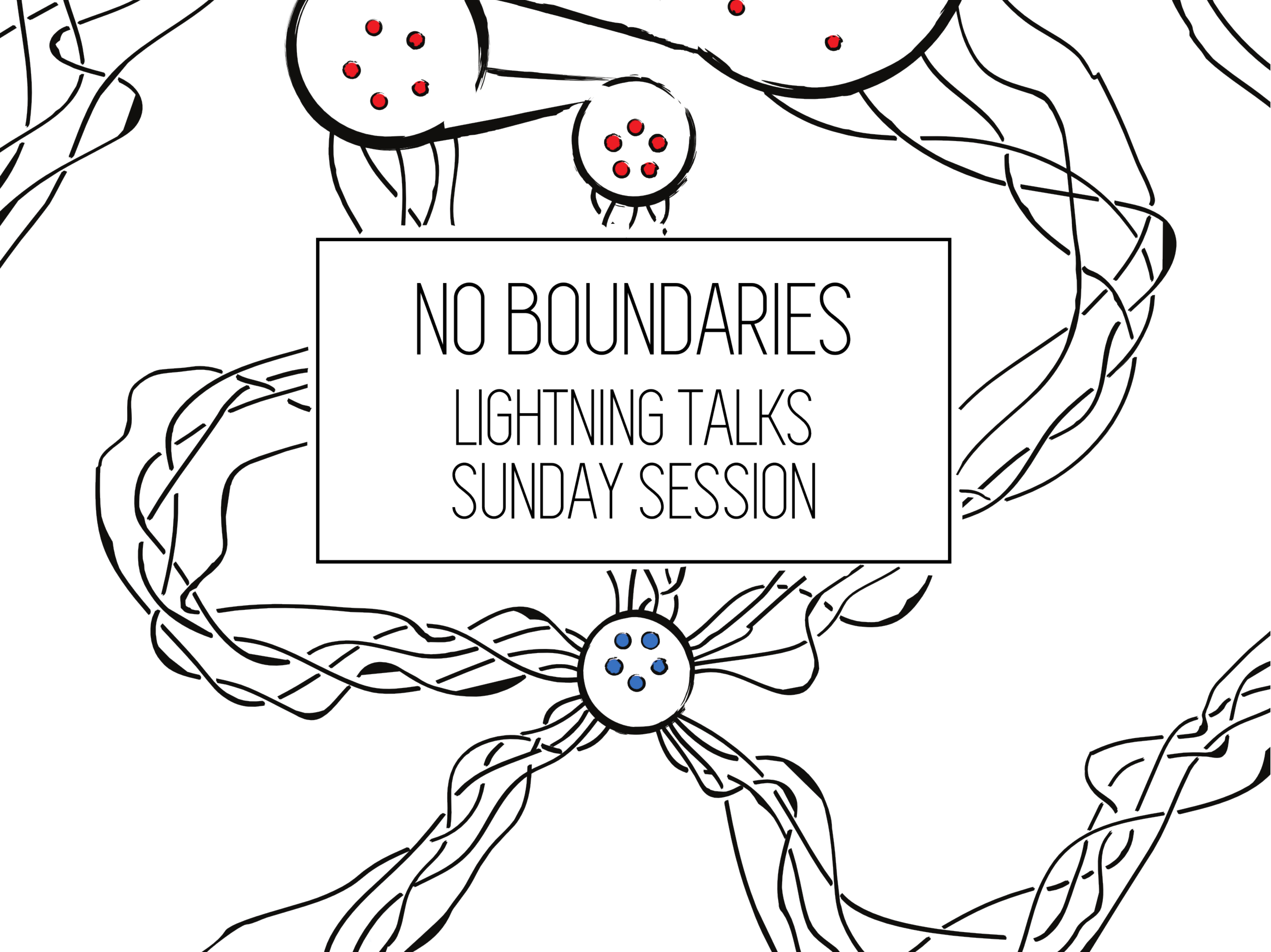
- Similar question for other enumerative problems.

Thank you.


NO BOUNDARIES
LIGHTNING TALKS
SUNDAY SESSION

Arithmetic Quotients:
of $\mathrm{Out}(F_n)$, $\mathrm{Mod}(\Sigma)$

Justin Malestein
(University of Oklahoma)

(includes work joint with Putman, and Grunewald–Larsen–Lubotzky)

Classical representations (or arithmetic quotients)

$$\mathrm{Out}(F_n) \twoheadrightarrow \mathrm{GL}_n(\mathbb{Z})$$

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Other representations?

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Other representations?

One possibility is to act on H_1 of finite covers (or of finite index subgroups of F_n)

Some Results, I

From actions on H_1 of finite index subgroups of F_n, one can obtain

Theorem (Grunewald–Lubotzky)

Let $n \geqslant 4$ and $m \geqslant 1$. There are virtual surjective representations $\mathrm{Out}(F_n) \to \mathrm{PGL}_{m(n-1)}(\mathcal{O})$. where $\mathcal{O}$ can be

- $\mathbb{Z}$
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- $\mathbb{Z}$
- *a ring of integers in a finite abelian extension of $\mathbb{Q}$ (depending on m)*
- *an order in a finite-dimensional division algebra over $\mathbb{Q}$ (not all such division algebras and can depend on m)*

Some Results, II

From actions on H_1 of finite covers of Σ_g, one can obtain

Theorem (Grunewald–Larsen–Lubotzky–M)

For any $g \geqslant 2, m \geqslant 1, n \geqslant 3$, $\exists$ *virtual surjections of* $\mathrm{Mod}(\Sigma_g)$ *onto:*

(a) $\mathrm{Sp}(2m(g-1), \mathbb{Z})$

(b) $\mathrm{Sp}(4m(g-1), \mathcal{O})$ *where* $\mathcal{O}$ *is the ring of integers in* $\mathbb{Q}(\zeta_n)^+$.

(c) $\mathrm{SU}(m(g-1), m(g-1), \mathbb{Z}[\zeta_n])$.

(d) *arithmetic groups of type* $\mathrm{SO}(2m(g-1), 2m(g-1))$.

ζ_n is a primitive nth root of unity.
This list is not exhaustive.

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Looijenga earlier found virtual surjective representations $\mathrm{Mod}(\Sigma_g) \to \mathrm{SU}(g-1, g-1, \mathbb{Z}[\zeta_n])$.

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Determining the (virtual) image in $\mathrm{Aut}(H_1)$ for a general finite cover is still open.

A Couple Potential Applications

A result of Putman–Wieland says: if nonzero $\mathrm{Mod}(\Sigma_g)$-orbits in H_1(cover) are always infinite for all finite covers, then $\mathrm{Mod}(\Sigma_g)$ cannot virtually map onto $\mathbb{Z}$ (otherwise it does)

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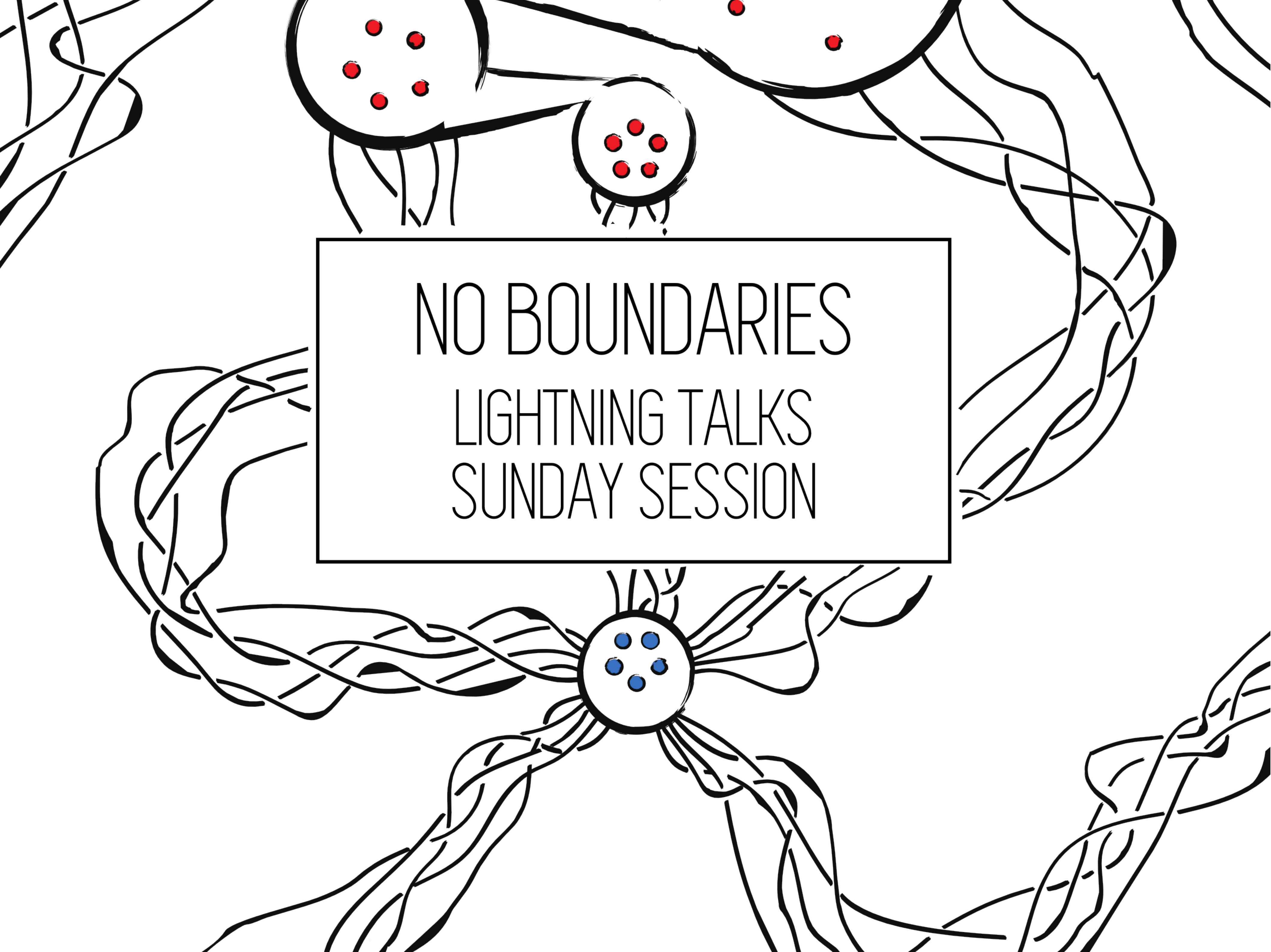
One can deduce facts about $\mathrm{Out}(F_n)/\langle \text{transvections}^k \rangle$ using results of M–Putman.


NO BOUNDARIES
LIGHTNING TALKS
SUNDAY SESSION

Stability in the Homology of Configuration Spaces

Jenny Wilson (Stanford)
joint with Jeremy Miller (Purdue)

No Boundaries: Groups in Algebra, Geometry, and Topology
27–29 October 2017

Configuration spaces

Definition (configuration space)

M – connected non-compact finite-type manifold of dim ≥ 2

$F_k(M)$ – (ordered) configuration space of M on k points

$$F_k(M) := \{(m_1, m_2, \ldots, m_k) \in M^k \mid m_i \neq m_j \text{ for all } i \neq j\}$$

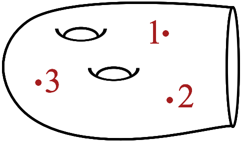


Figure: A point in $F_3(M)$

$$S_k \curvearrowright F_k(M)$$

Goal: Understand $H_*(F_k(M))$

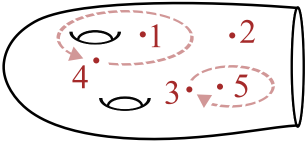


Figure: A class in $H_2(F_5(M))$

Representation Stability

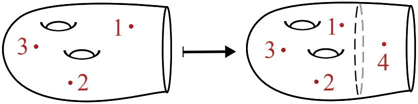


Figure: Stabilization Map $t : F_k(M) \to F_{k+1}(M)$

Strategy: Fix M. Package the sequence $\{H_*(F_k(M))\}_k$ into a module over a category encoding S_k–actions and embeddings.

Theorem (Church–Ellenberg–Farb, M–W (non-orientable case))

For each fixed i, $\{H_i(F_k(M))\}_k$ *is* **representation stable**.

$$\mathbb{Z}[S_{k+1}] \cdot t_*(H_i(F_k(M);\mathbb{Z})) = H_i(F_{k+1}(M);\mathbb{Z}) \qquad \textit{for } k \geq 2i.$$

Higher-Order Representation Stability

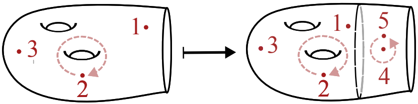


Figure: Secondary stabilization map $t' : H_i(F_k(M)) \to H_{i+1}(F_{k+2}(M))$

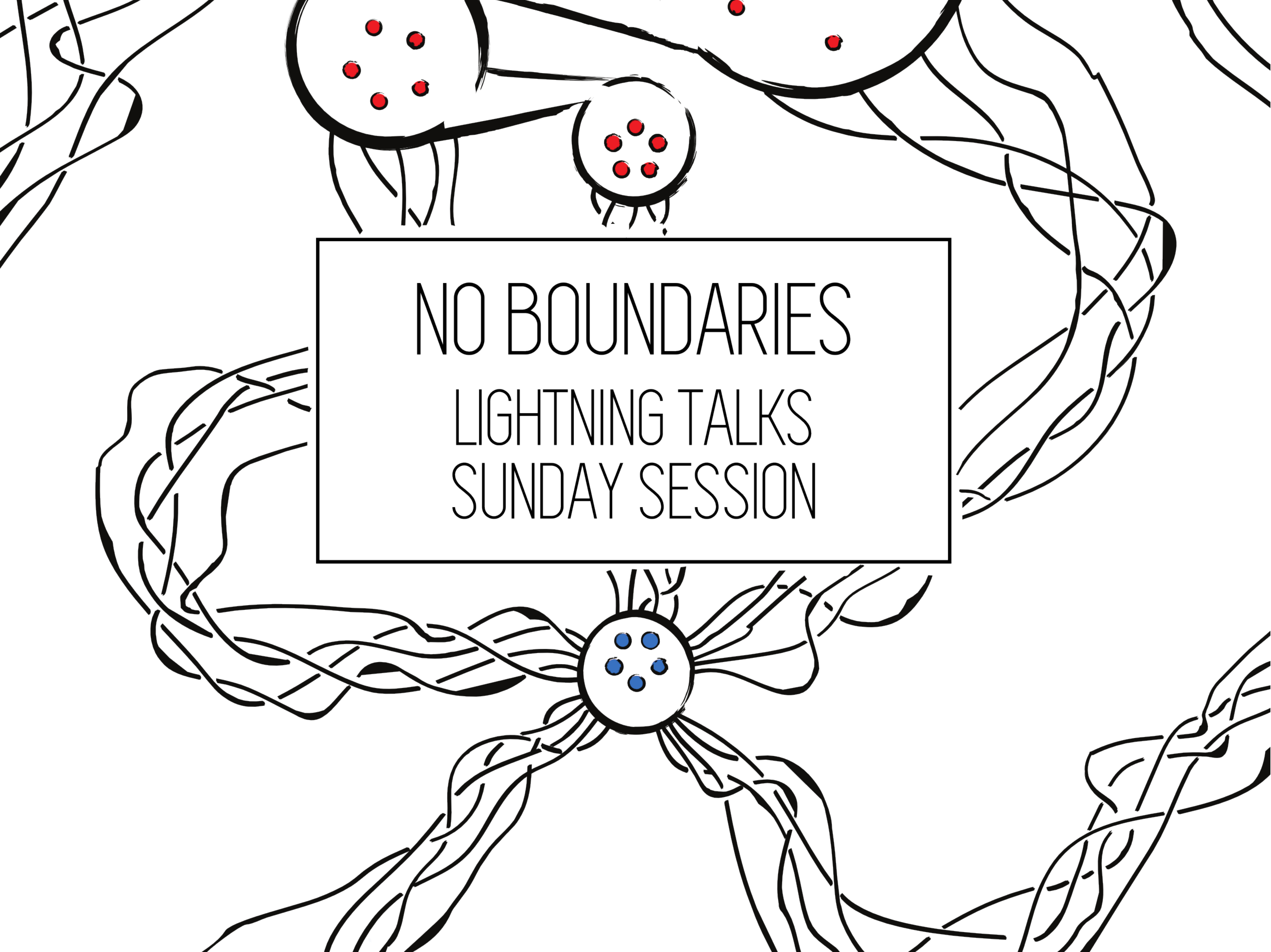
Theorem (M–W)

$\{H_*(F_k(M);\mathbb{Q})\}_k$ *has* **secondary representation stability**.

For each fixed i, the sequence of "unstable" homology in

$$\left\{H_{\frac{k+i}{2}}(F_k(M);\mathbb{Q})\right\}_k$$

is finitely generated under the actions of maps t' and the groups S_k.



NO BOUNDARIES

LIGHTNING TALKS
SUNDAY SESSION

Arithmetic groups and characteristic classes of manifold bundles

Bena Tshishiku
Harvard University

Dark Matter Problem

Dark Matter Problem

$$\mathrm{H}^*(\mathrm{Mod}(S_g); \mathrm{Q})$$

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$$\mathrm{H}^*(\mathrm{BDiff}(S_g);\ \mathbb{Q}) \simeq \mathrm{H}^*(\mathrm{Mod}(S_g);\ \mathbb{Q})$$

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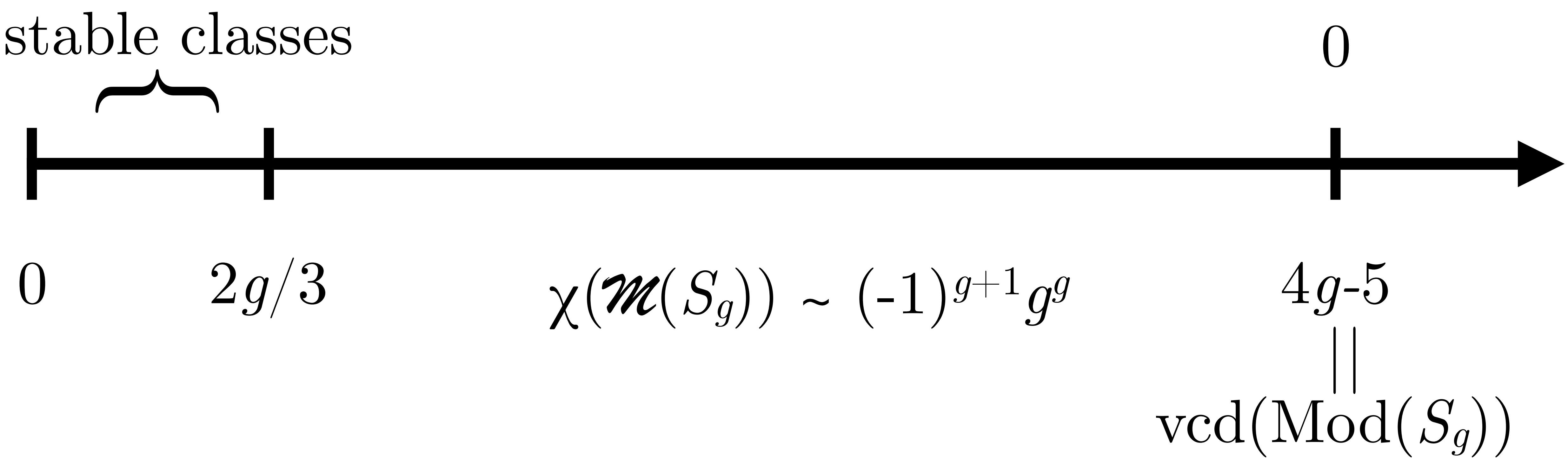

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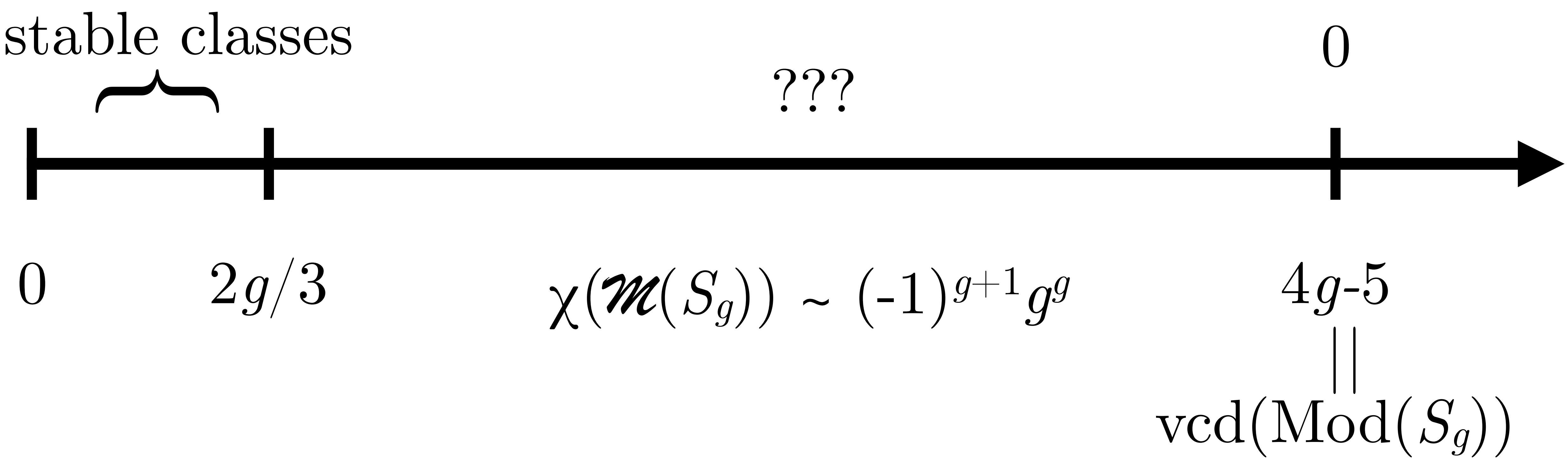
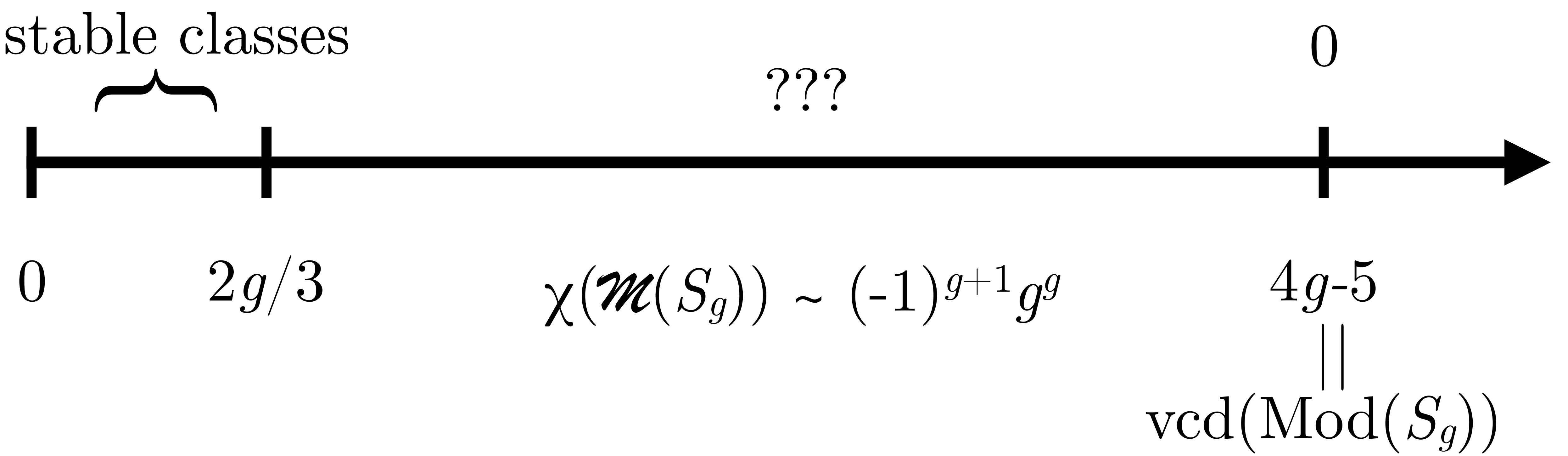


Dark Matter Problem: Find new classes outside the stable range. (e.g. classes in odd degree??)

$$H^*(\mathrm{BDiff}(S_g);\ Q) \simeq H^*(\mathrm{Mod}(S_g);\ Q) \simeq H^*(\mathcal{M}(S_g);\ Q)$$



Characteristic classes of real vector bundles with arithmetic structure group

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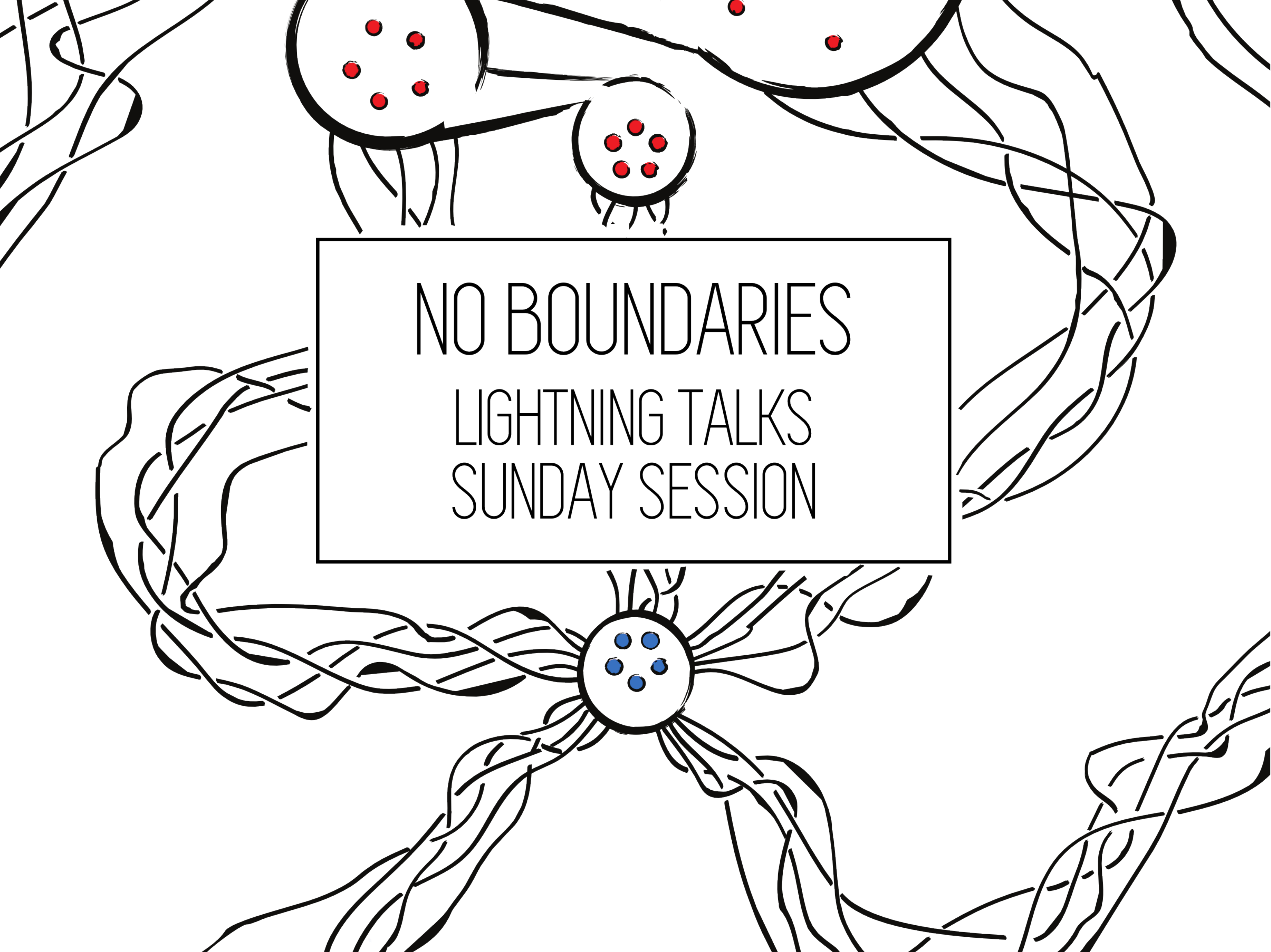
Happy birthday, Benson!


NO BOUNDARIES
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SUNDAY SESSION

Groups ... in Other Places

Angela Kubena

Department of Mathematics
University of Michigan

No Boundaries

29 Oct 2017

As we all know, examples are important...

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In this talk : An example from undergraduate education

Background: Characteristics of Calculus at Michigan

Single Variable Calculus

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Multivariable Calculus

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Multivariable Calculus

- Large lecture
  - plus group work in lab
- $\sim 64\%$ of the students are from Engineering
- "Standard" focus
- Applications From Physics
- Covers Stokes and Divergence Theorems

This multivariable calculus course is NOT ideal for many students.

A New Course

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Course begins January 2018

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