Twists of the Rabbit Polynomial

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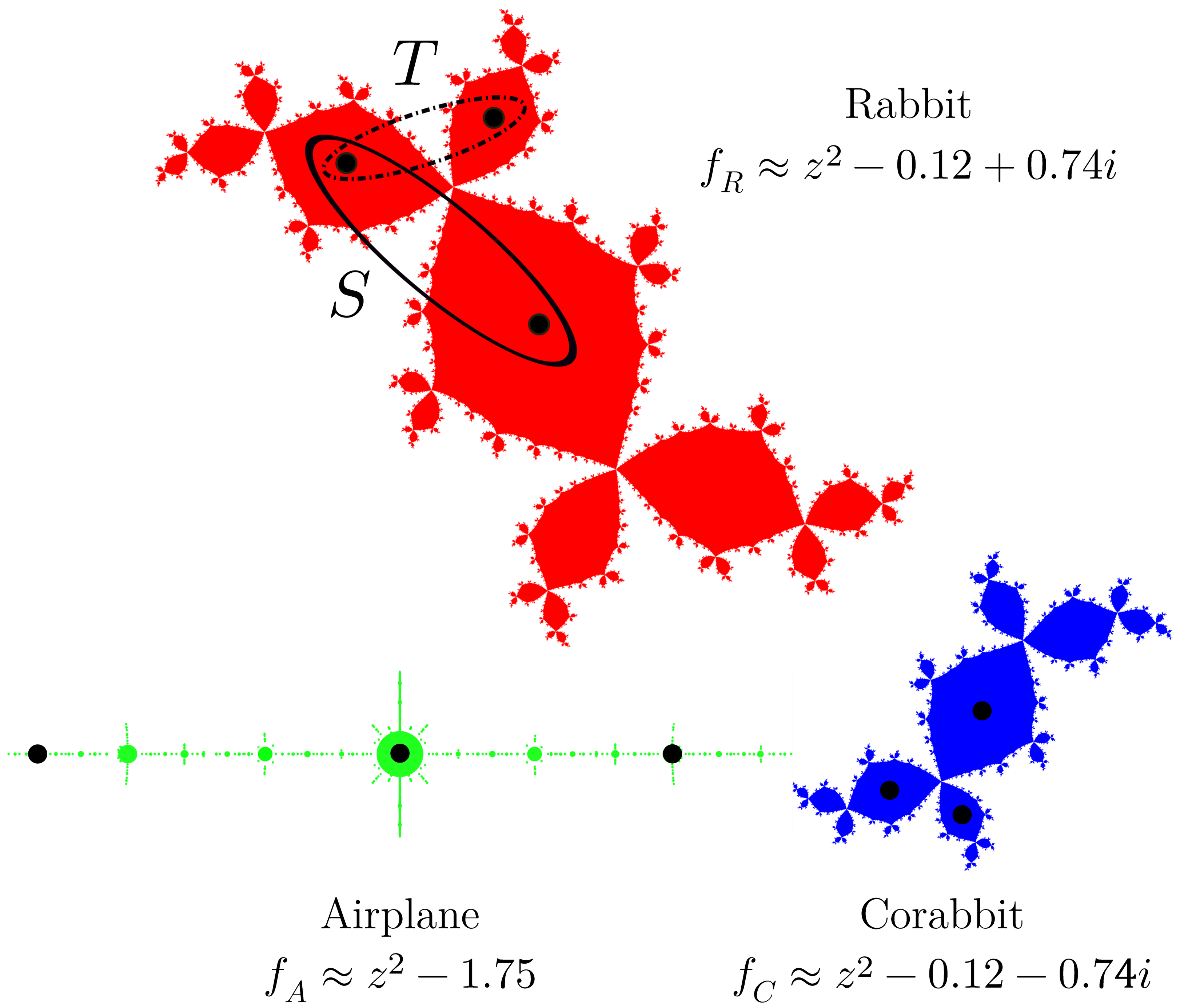
Three Polynomials

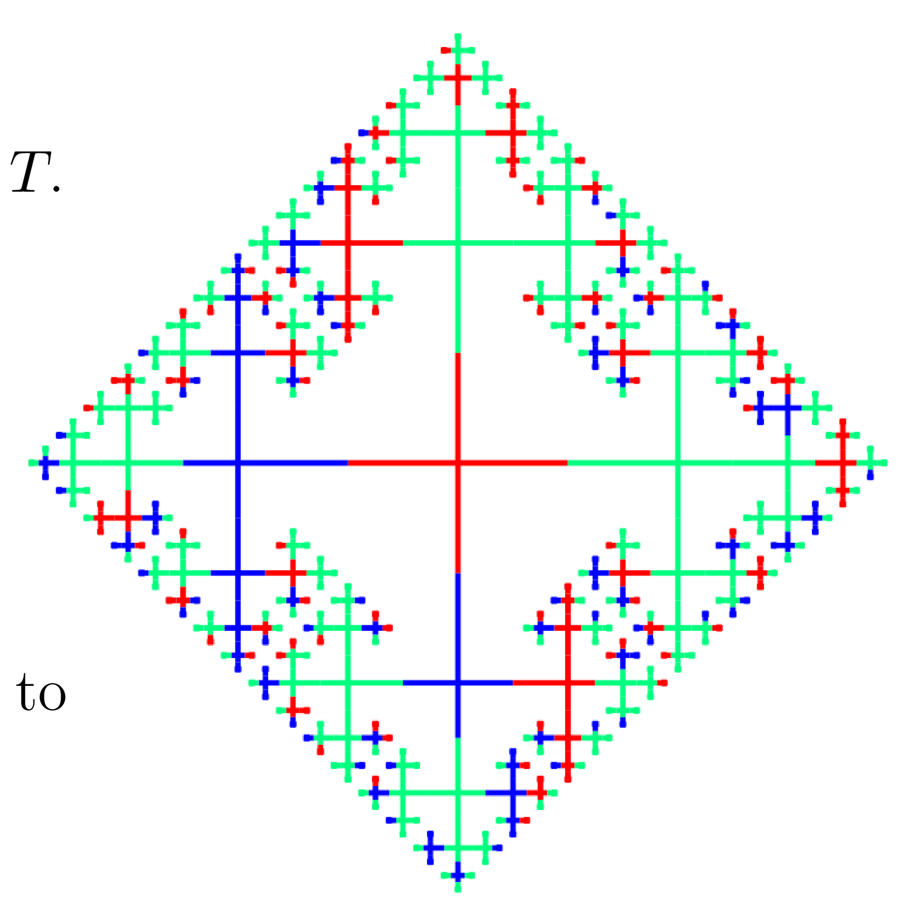


Twisted Rabbit Problem

Let g be a product in the Dehn twists about S and T. Since S and T generate a free group, we have:

$$g \in \langle S, T\rangle \cong F_2$$

Fact: $f_R \cdot g$ is equivalent to one of f_R, f_C, or f_A.

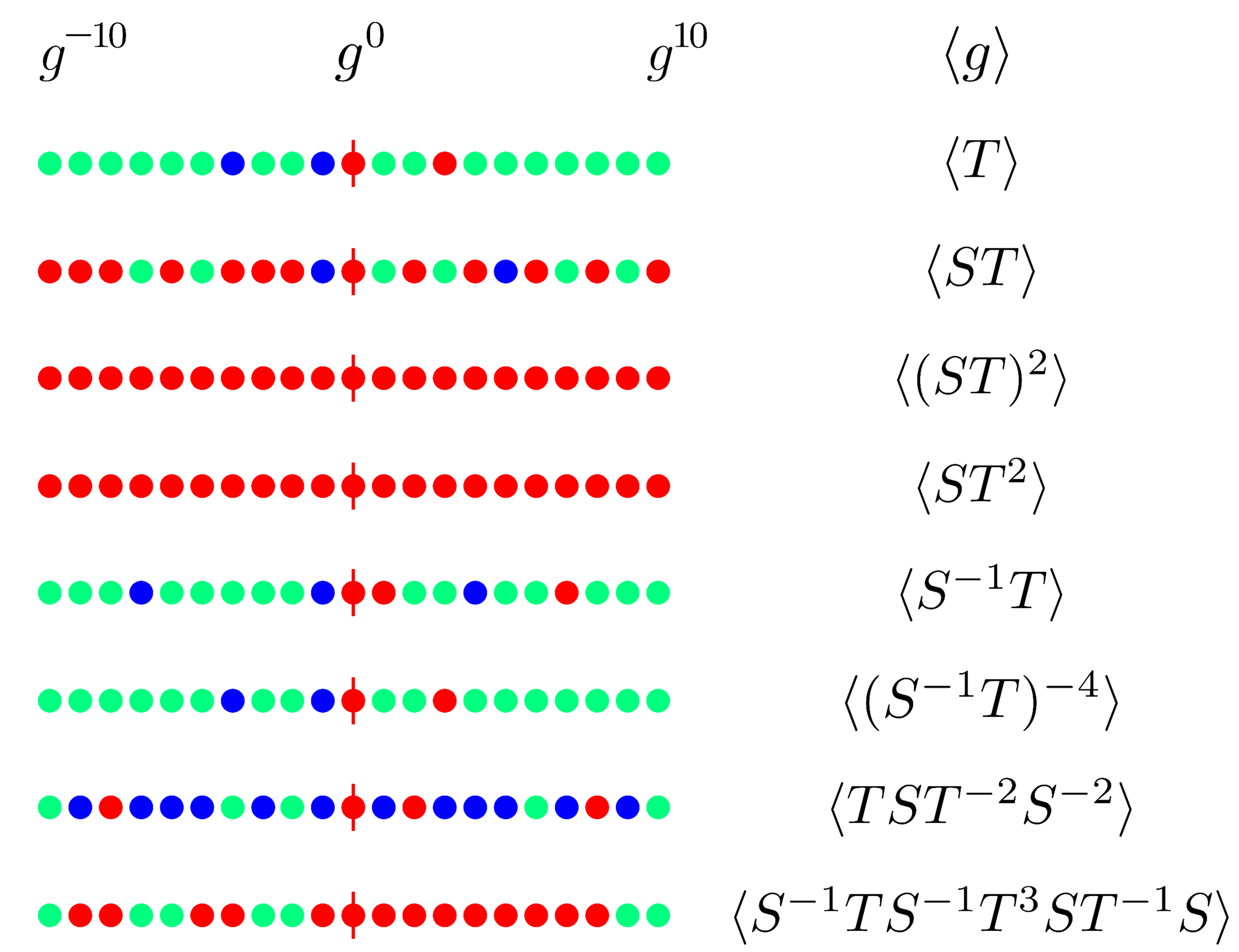


Bartholdi and Nekrashevych (2006) gave an algorithm for determining which one.

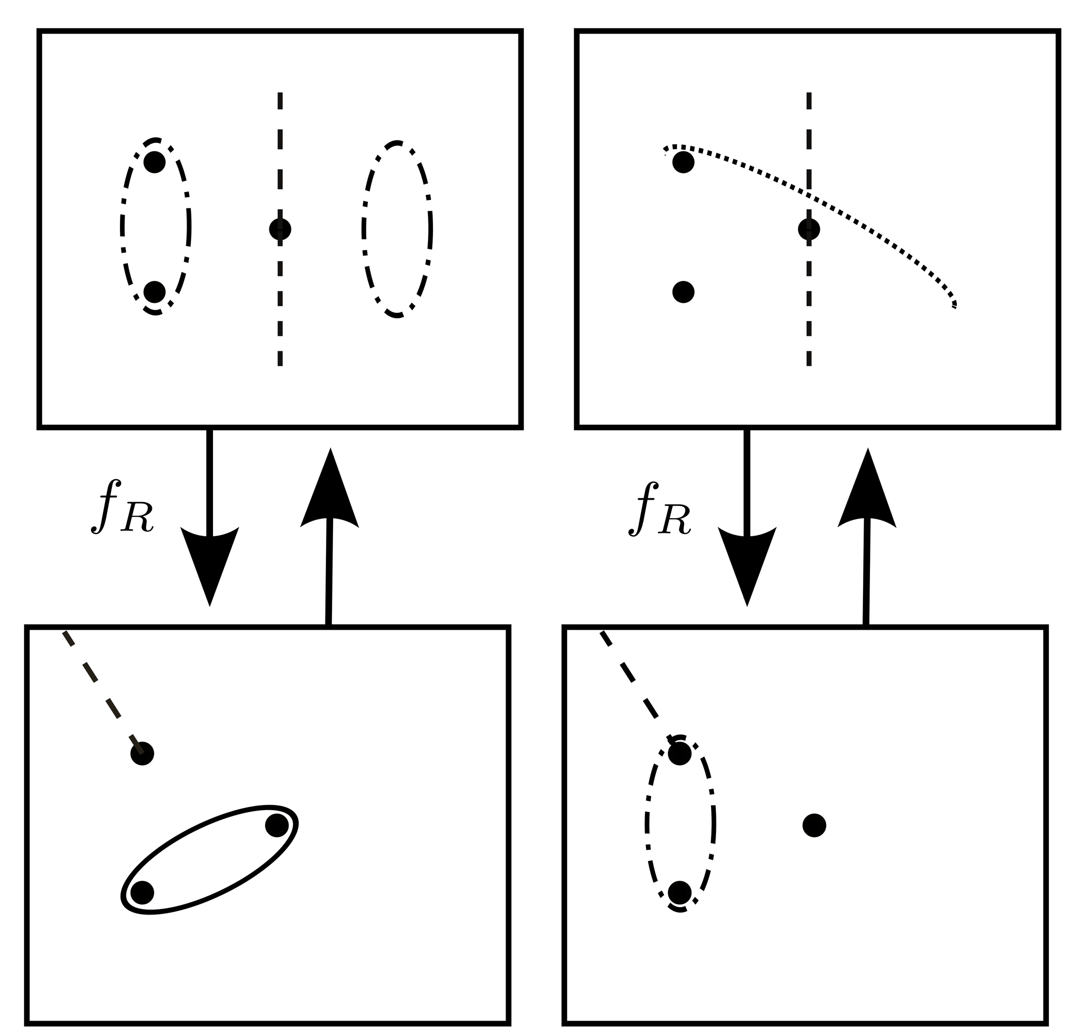
Question

Are there all-rabbit subgroups of $\langle S, T\rangle$?

Examples of Subgroups



Method: Reduction by Lifting



S lifts to T T does not lift

Sample Calculations

g	lift(g)
S	T
T^2	$S^{-1}T^{-1}$
TST^{-1}	1

Fact: $f_R \cdot g$ is equivalent to $f_R \cdot \text{lift}(g)$ when g is liftable.

$$\begin{aligned} ST^2 &\sim \text{lift}(ST^2) \\ &= \text{lift}(S)\text{lift}(T^2) \\ &= T \cdot S^{-1}T^{-1} \\ &\sim \text{lift}(TS^{-1}T^{-1}) \\ &= 1 \end{aligned}$$

Fact: $f_R \cdot g$ is equivalent to $f_R \cdot T\text{lift}(gT^{-1})$ when g is unliftable.

$$\begin{aligned} S^{-1}T &\sim T{\cdot}\text{lift}(S^{-1}T \cdot T^{-1}) \\ &= T \cdot T^{-1} \\ &= 1 \end{aligned}$$

Results

1. No non-trivial subgroup of $\langle T\rangle$ is all-rabbit.
2. Every infinitely liftable twist generates an all-rabbit subgroup.
3. In fact, the collection of all infinitely liftable twists generates an all-rabbit subgroup.
4. We developed a program that computes lifts of twists and creates visuals of subgroups.
5. No unliftable twist of word length 12 or less generates an all-rabbit subgroup.
6. We conjecture that no unliftable twist generates an all-rabbit subgroup.

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