The Curve Graph of the 5-Punctured Sphere

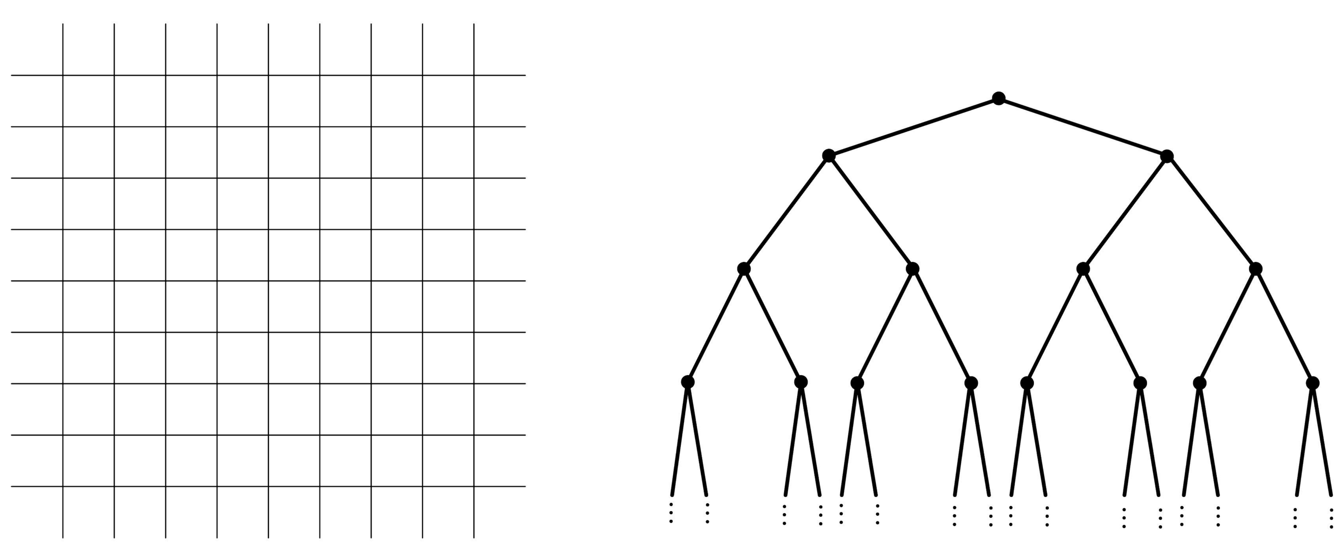
Sami Aurin[1] and Darrion Thornburgh[2]   Mentors: Wade Bloomquist[1] and Dan Margalit[1]

Our Project

Hensel, Przytycki, and Webb proved the curve graph of a surface is 17-hyperbolic. Our goal is to show it is **not** 1-hyperbolic for the 5-punctured sphere.

Hyperbolicity

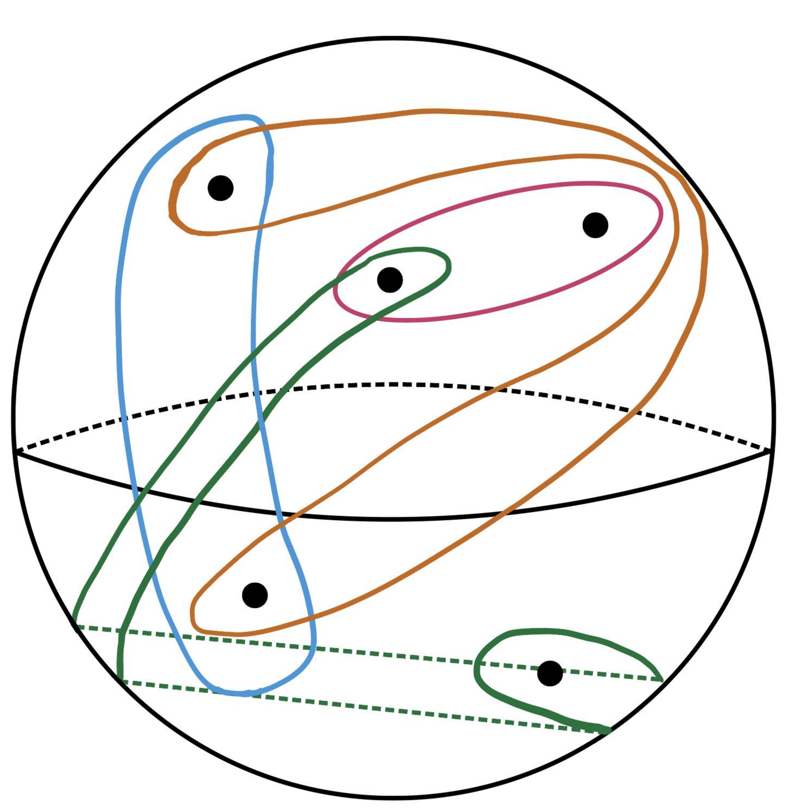
Hyperbolic space comes in many forms: trees, the hyperbolic plane, curve graphs of surfaces, etc.



Euclidean space is not hyperbolic but a tree is.

Curve Graph $C(S)$

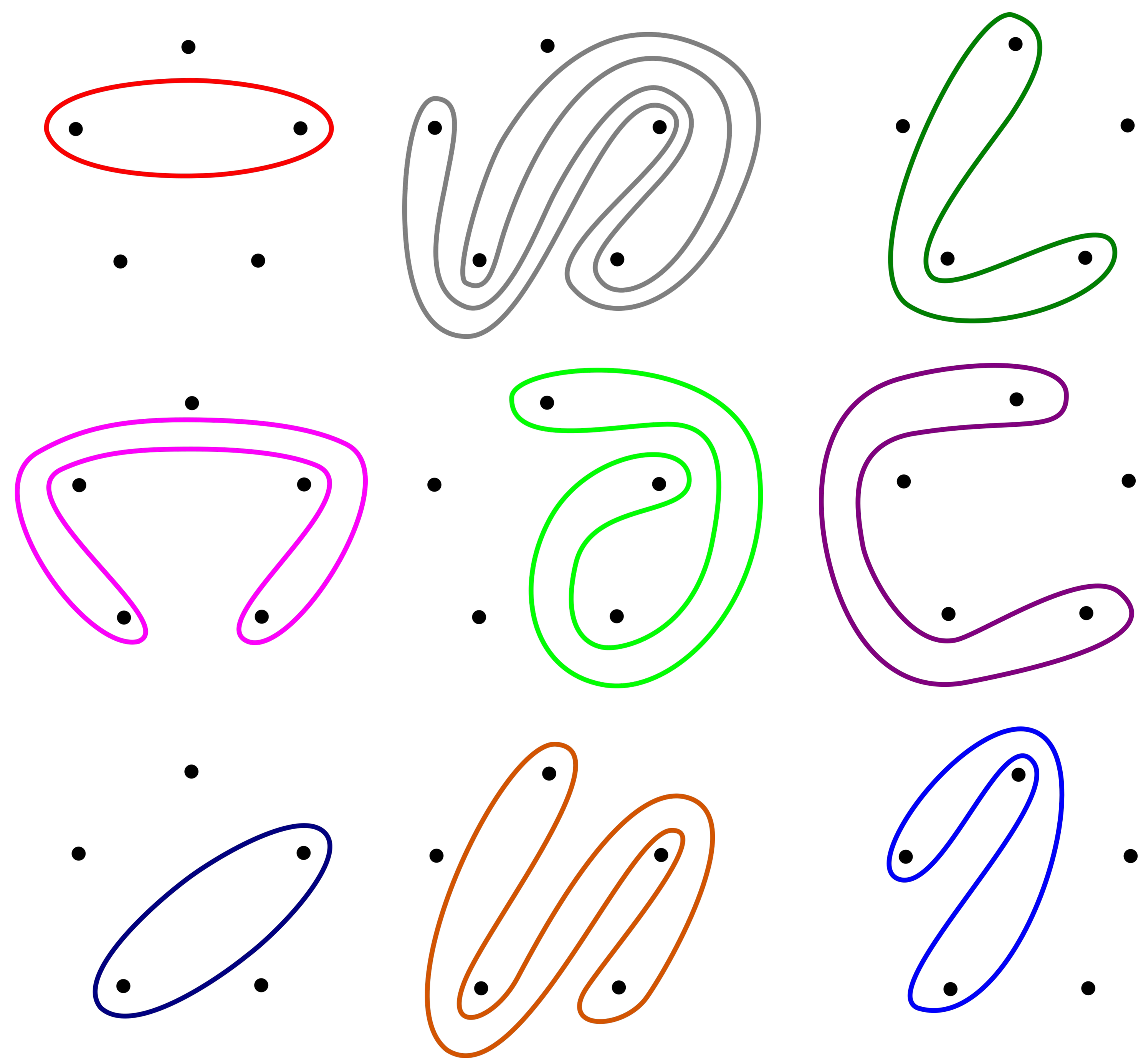
The **curve graph** $C(S)$ of a surface S is the graph where vertices are curves and the edges represent disjointedness.



A geodesic path of length 3 in $C(\Sigma_{0,5})$.

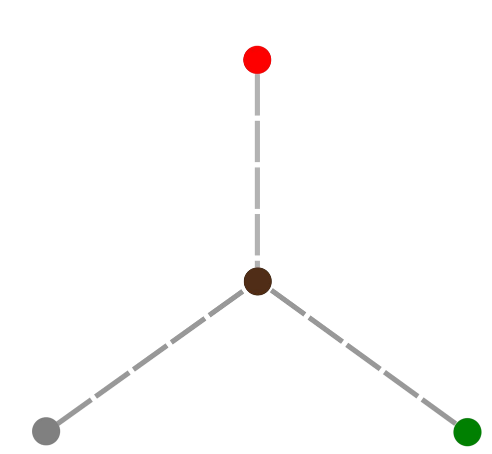
$C(\Sigma_{0,5})$ is not 1-hyperbolic

Theorem (Aurin-Thornburgh): The following curves form a geodesic triangle that is not 1-centered.

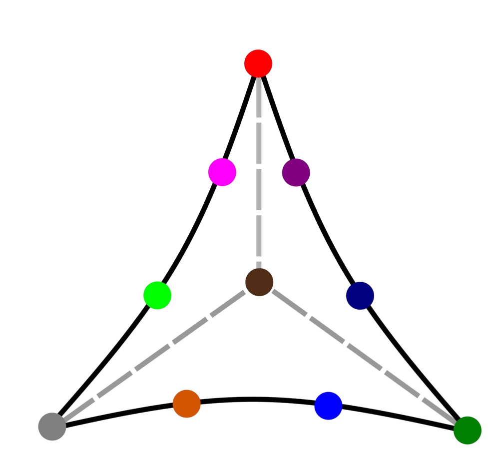


Centered Triangles

A triangle in $C(S)$ is δ**-centered** if there exists a vertex that is at most δ away from each side. If all triangles are δ-centered, then $C(S)$ is δ**-hyperbolic**.



0-centered triangle

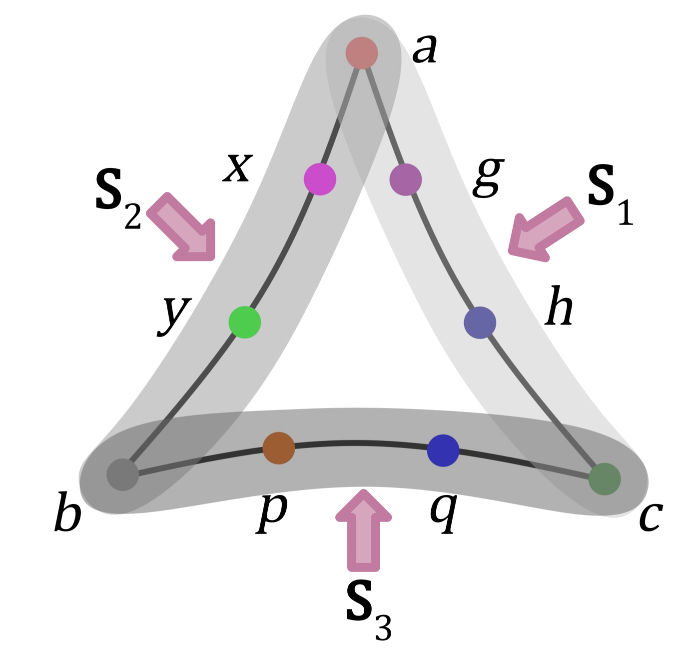


1-centered triangle

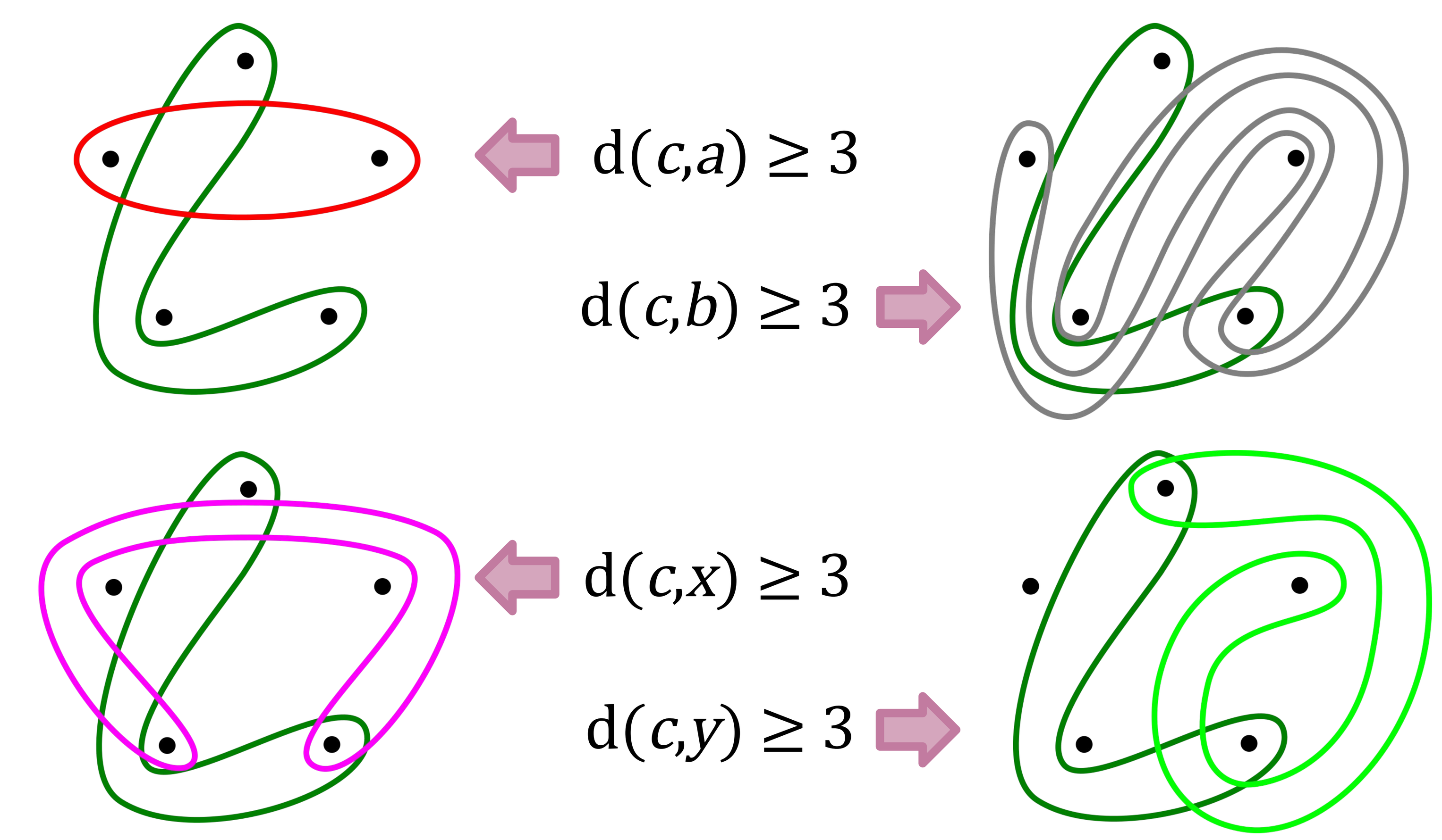
Not 1-Centered

Lemma: A geodesic triangle is not 1-centered if the following hold:

1.) $\min\{d(b,\alpha) : \alpha \in S_1\} \geq 3$;
2.) $\min\{d(c,\alpha) : \alpha \in S_2\} \geq 3$;
...



Checking Condition 2



Future Work

Extending our result to $C(\Sigma_{0,p})$ for all p ≥ 6 and possibly to surfaces with genus.

Acknowledgments

We would like to thank our mentors Dr. Wade Bloomquist and Dr. Dan Margalit. We would also like to thank the NSF for funding this research.

1: Georgia Institute of Technology   2: Bard College