

Genus-0 Virtual Braids



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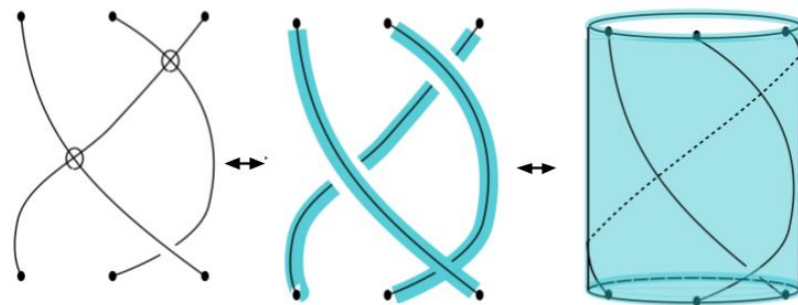
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Overview

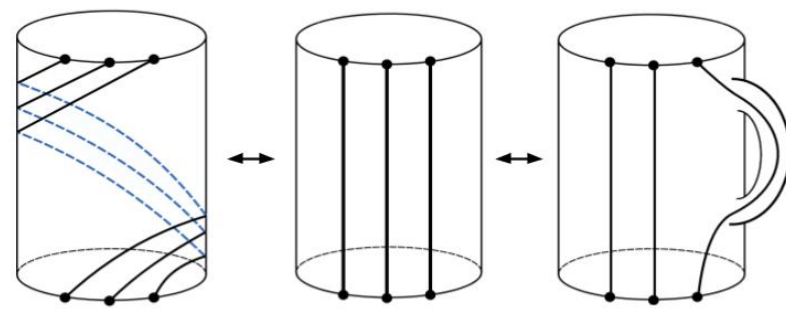
The virtual braid group extends the classical braid group by the symmetric group. We define a new subgroup of virtual braids using a topological property, and we investigate its algebraic structure.

Virtual Braids

There is a bijection between virtual braids and **thickened surface diagrams**.



Thickened surface diagrams are equivalent up to **homeomorphism** and **stability** moves.



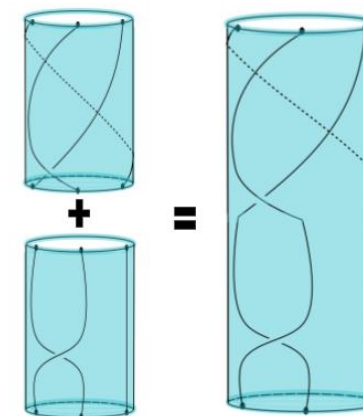
The genus of a braid is the minimal genus of its thickened surface diagram. The braid above has **genus 0**.

New subgroup of VB_n

Definition. VB_n^0 is the set of all genus-0 virtual braids.

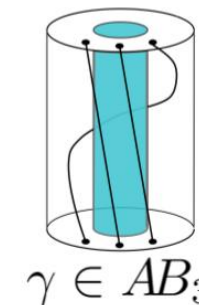
Proposition 1. The bijection between virtual braids and thickened surface diagrams is an isomorphism.

Proposition 2. VB_n^0 is a subgroup of the virtual braids.

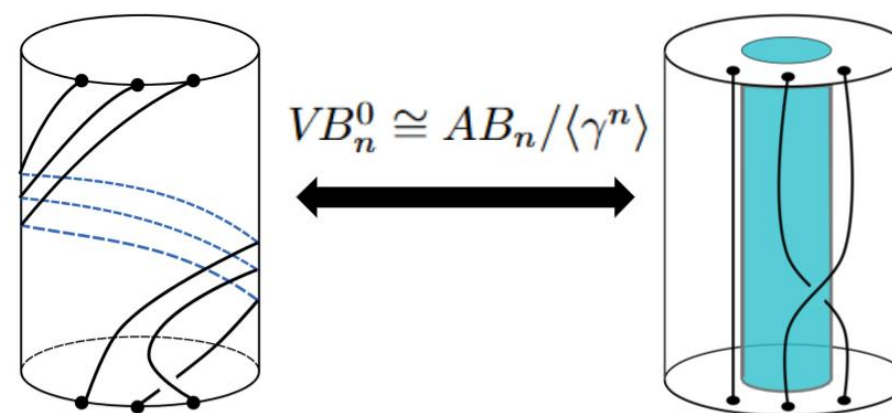


The operation on thickened surface diagrams is **stacking**.

Question: Are genus-0 braids annular braids?



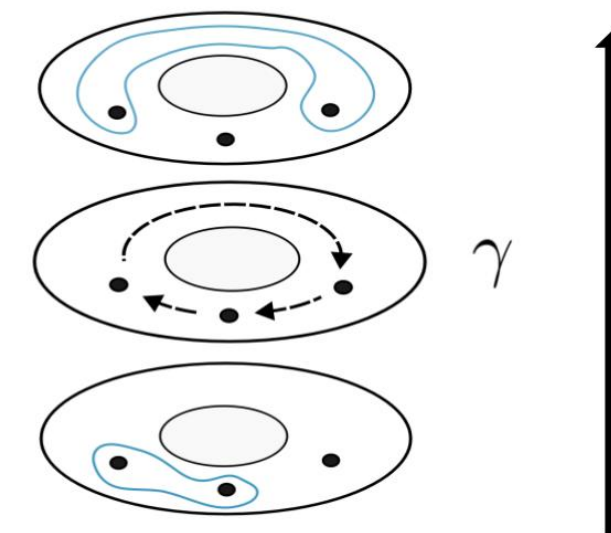
Main Theorem



Corollaries

These results follow from facts known about the annular braid group:

- VB_n^0 acts on curves on the annulus
- VB_n^0 is a subgroup of $\text{Aut}(F_n)$
- VB_n^0 is linear



Future Work

- Extend action on curves for genus greater than 0.
- Compute bounds on genus of a given virtual braid.