Coloring the Curve Complex with Rabbits, Corabbits, and Airplanes

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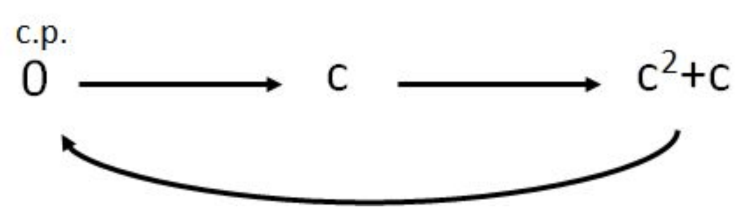
Background

Rabbit polynomial:

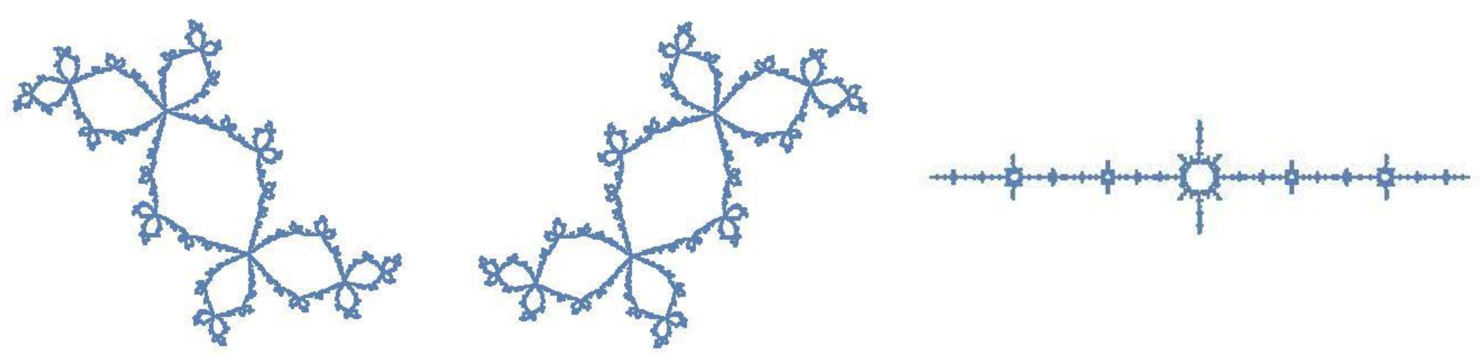
$$R(z) = z^2 + c$$

$$c \approx -0.1226 + 0.7449i$$

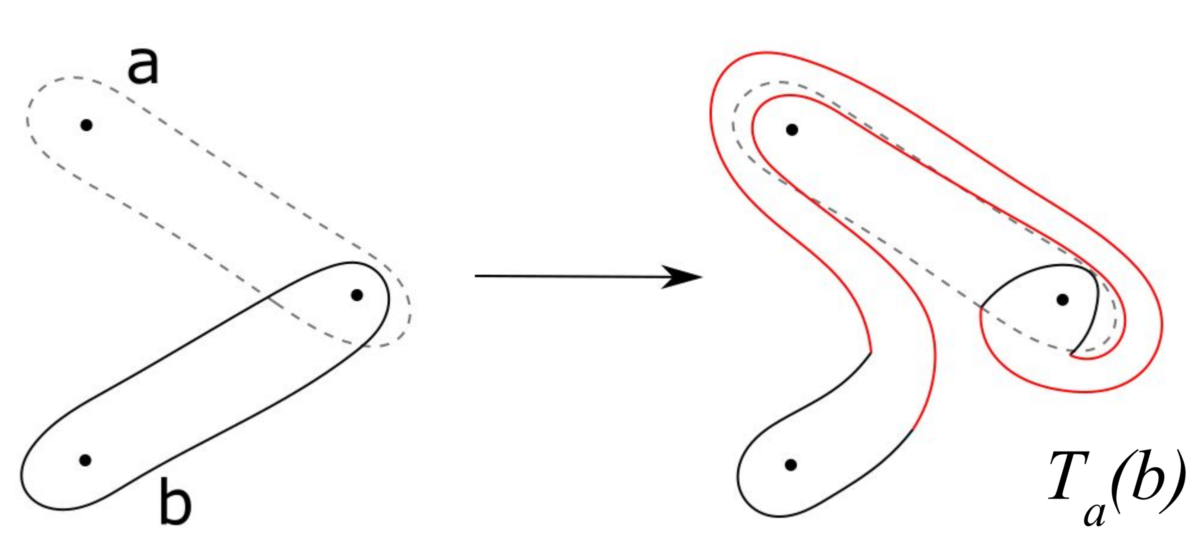
Periodicity:



Rabbit Corabbit Airplane



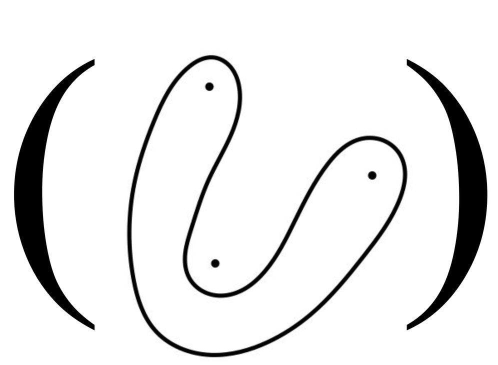
Dehn Twist



For any curve d, $T_d \circ R$ is equivalent to one of the rabbit, corabbit, airplane.

$$f : \{\text{curves}\} \to \{R, C, A\}$$

Example:

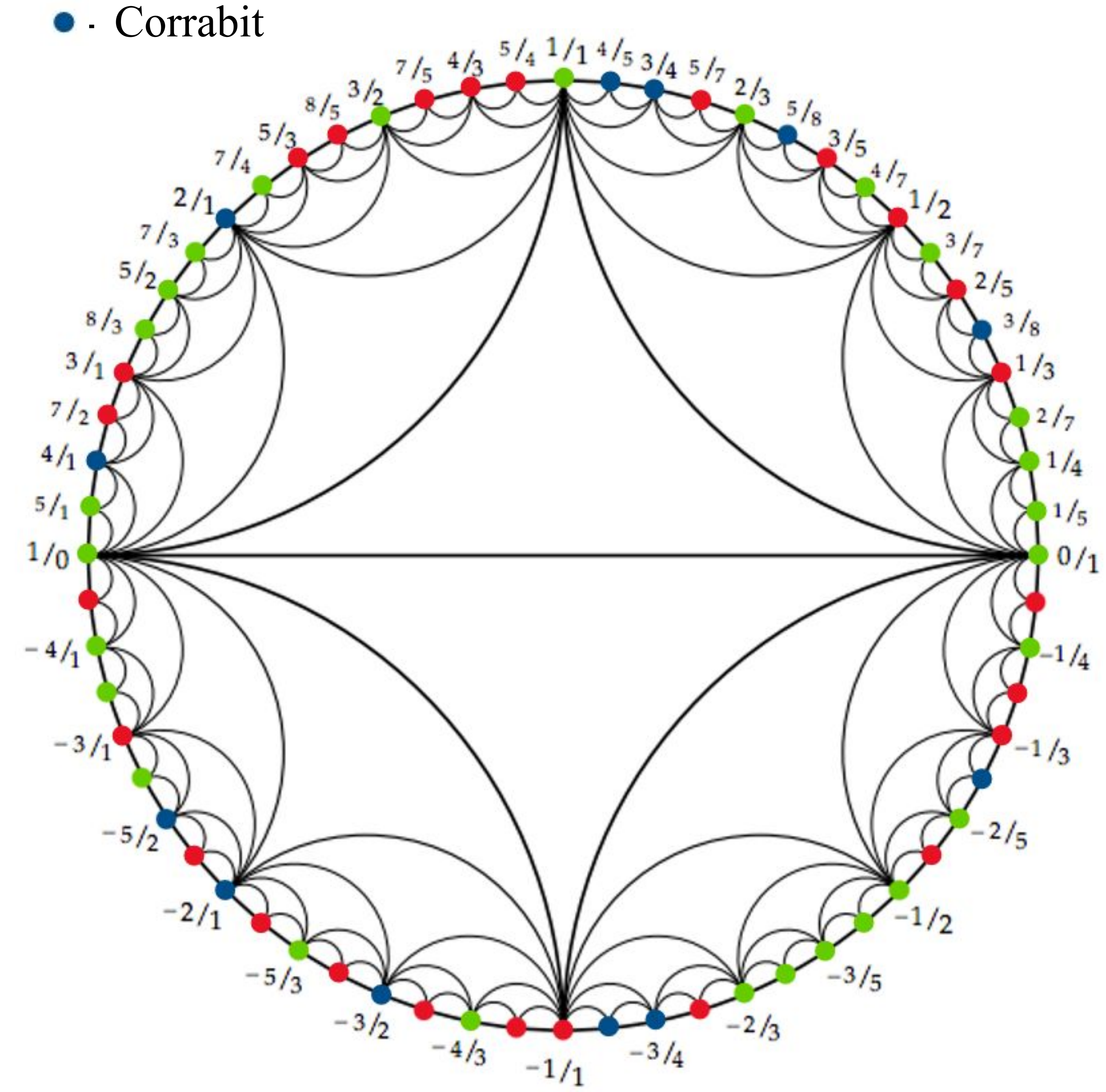
f() $= C$

Goal

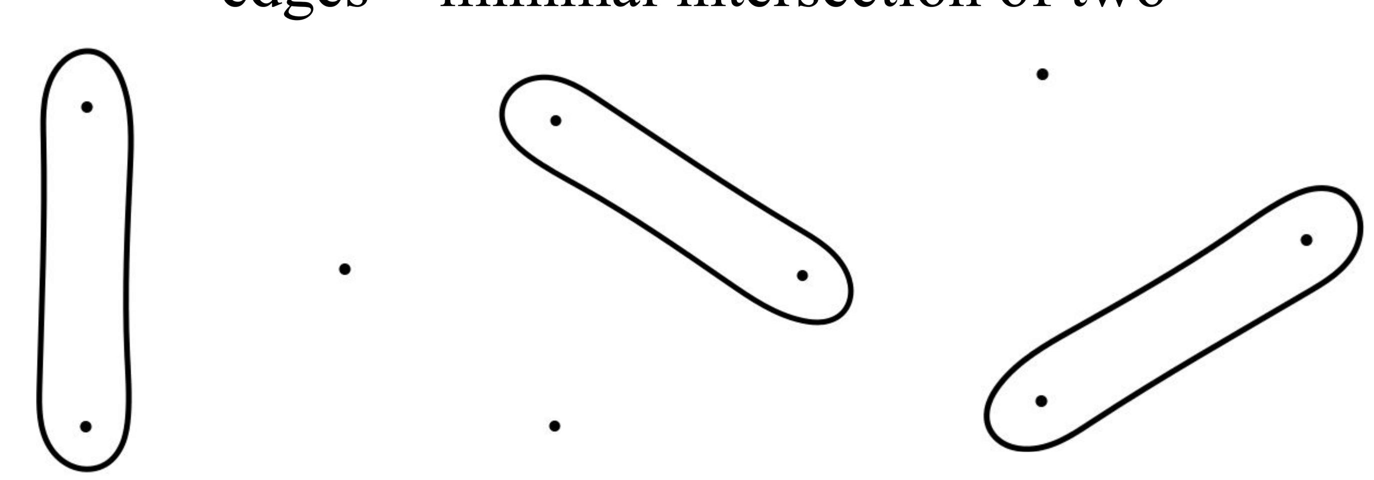
Given a curve d, we want to determine $f(d)$.

Curve Complex

- Airplane
- Rabbit
- Corrabit



vertices = curves
edges = minimal intersection of two

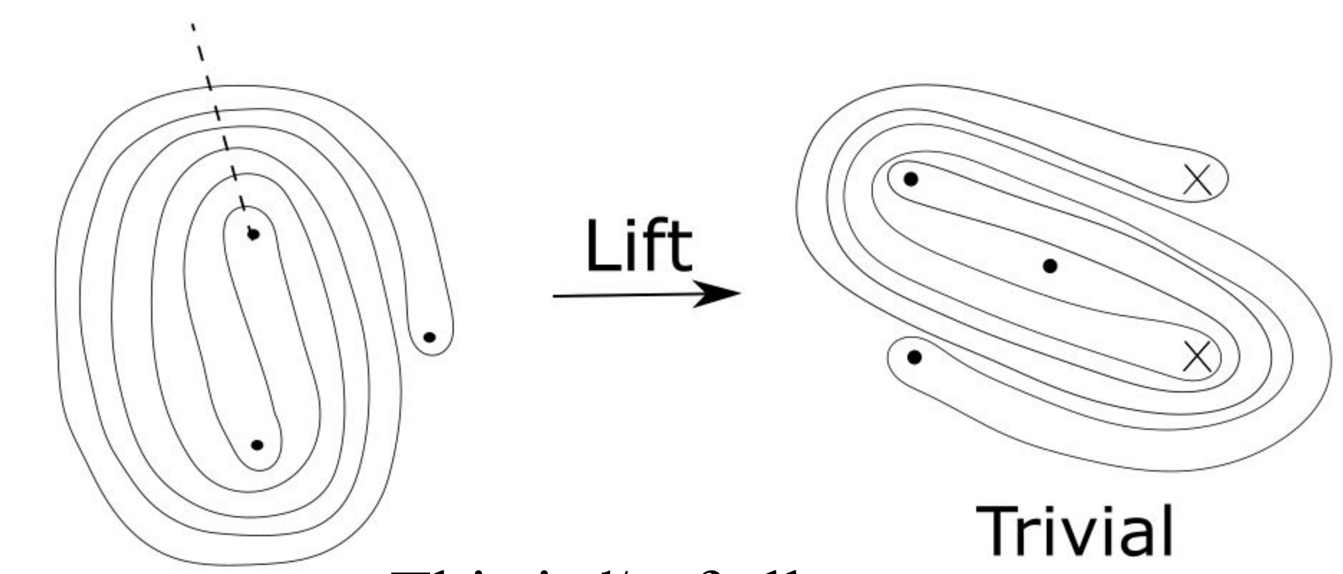


(1/0) (0/1) (1/1)

Infinitely Many Rabbits!

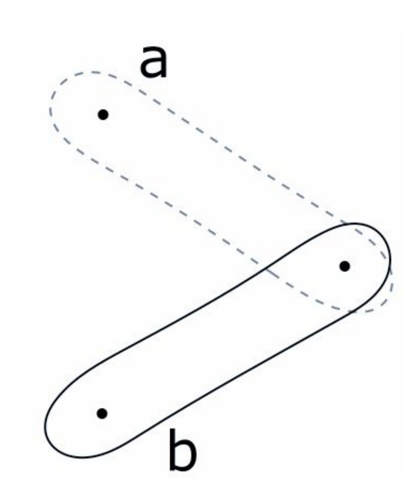
For any liftable curve d that intersects the dashed ray 2 mod 4 times, the lift of d is trivial, hence $f(d)=R$.

Example:



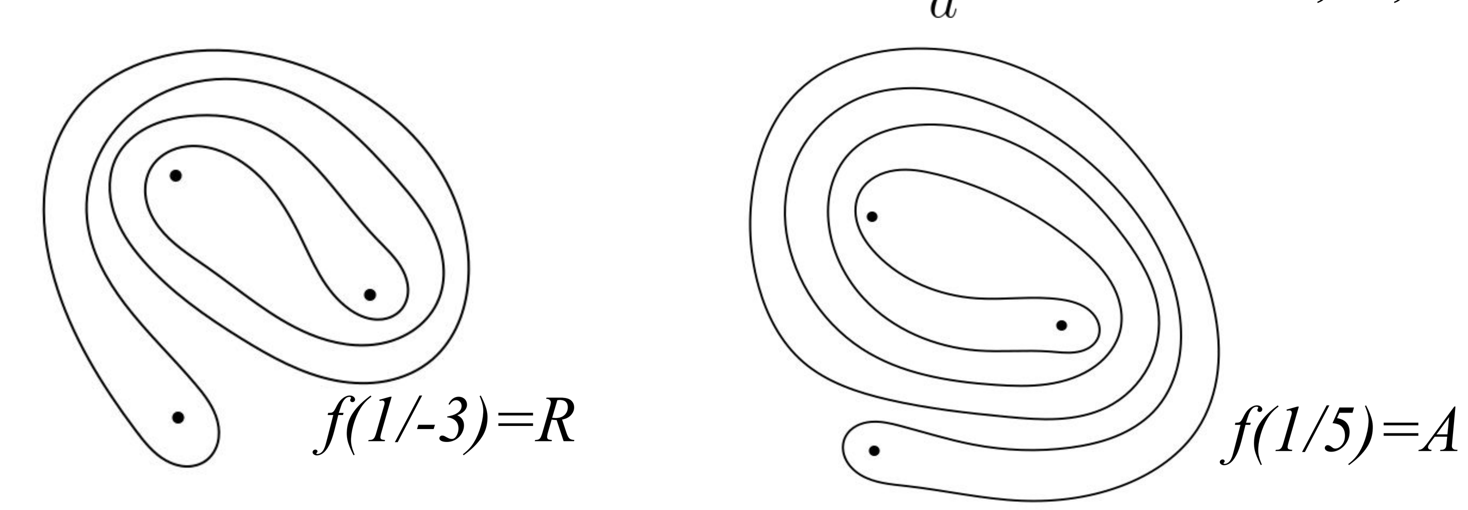
This is ⅛ of all curves.

Other Infinite Families



$$f(1/1-2k) = f(T_a^k(b))$$

- k is odd: $f(1/1-2k) = R$
- k=2m : $f(1/1-2k) \simeq T_a^{1-m} \circ R$
  - Bartholdi-Nekrashevych algorithm determines $T_a^{1-m} \circ R$ as R, C, or A



Acknowledgments

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