Finite Subgraphs of the Curve Graph

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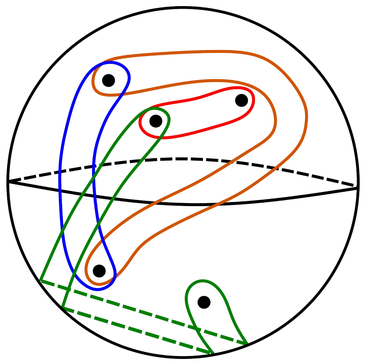
Our Project

The curve graph of a surface is an infinite graph with a rich structure. We aim to study the curve graph of 5-punctured sphere by characterizing its finite subgraphs.

Curve Graph $C(S)$

The **curve graph** $C(S)$ of a surface S is the graph where vertices are curves and the edges represent disjointedness.



A path of length 3 in $C(\Sigma_{0,5})$

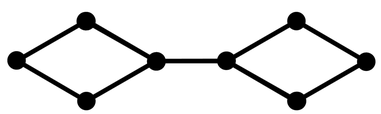
A graph is **realizable** on a surface if there exists a collection of curves on the surface which are disjoint if and only if the two curves share an edge.

Results

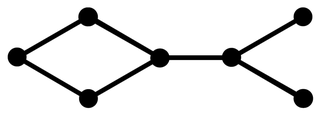
We found **no simple characterization** in terms of a finite list of primitive unrealizable graphs. We were able to characterize some restrictions with a graph homomorphism.

Hereditary Property

A hereditary graph property is a property which is inherited by all induced subgraphs.



A triangle-free graph G.

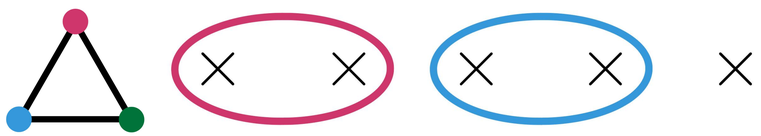


G - {v} is also triangle-free.

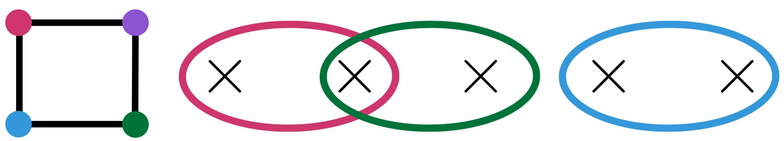
In the curve graph,- taking induced subgraphs corresponds to deleting curves.

Primitive Graphs

A **primitive unrealizable graph** is an unrealizable for which every induced subgraph can be realized.



The triangle is a primitive unrealizable pattern.



The square is a primitive unrealizable pattern.

The most critical primitive unrealizable graphs we found were the 4-cycle and the Petersen graph.

Curve Graph Homomorphism

Using homology, we obtain a graph homomorphism $f: C(\Sigma_{0,5}) \to P$. This can be generalized.

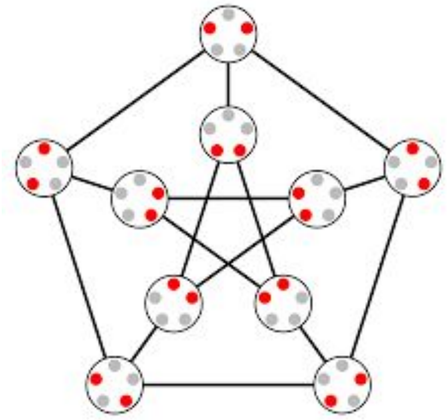


Image from Wikipedia.

Conjecture: There are finite number of primitive unrealizable graphs that have a homomorphism to P.

Future Work

Using forbidden patterns and homomorphisms to compute bounds on the chromatic number of the curve graph for surfaces with genus.

Acknowledgments

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1: Yale University 2: Brown University 3: Vanderbilt University 4: Georgia Institute of Technology