



# Dynamics of Mapping Class Groups

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## Background

How efficient is this taffy puller?



$n$ -armed taffy pulling action  $\leftrightarrow$   
 homeomorphism of an  $n$ -punctured plane

**Nielsen-Thurston Classification Theorem**  $\rightarrow$  to every homeomorphism of a surface we can attach a real number called the stretch factor

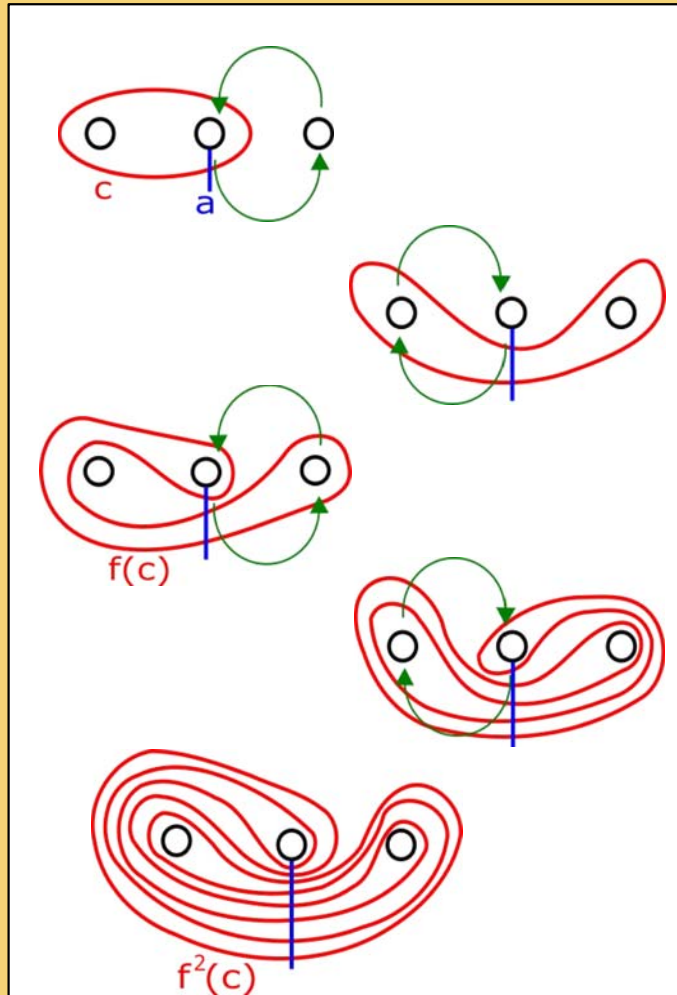
**Setup:**  $c$  = curve,  
 $f$  = homeomorphism,  
 $a$  = reference arc.

**Stretch Factor** = growth rate of  $i(f^n(c), a)$

**Margalit-Strenner-Yurttas:** Quadratic time algorithm that computes the stretch factor.

**Our Project:** Implement the algorithm.

## Example

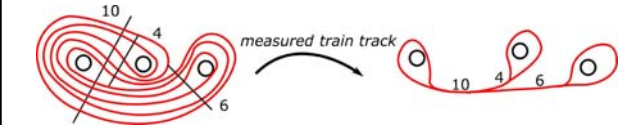


$$\text{Stretch Factor} = \phi^2 \approx 2.618$$

## The General Case

**Challenge:** How can we compute  $f^n(c)$  for arbitrary  $f$ ,  $n$ , and  $c$ ?

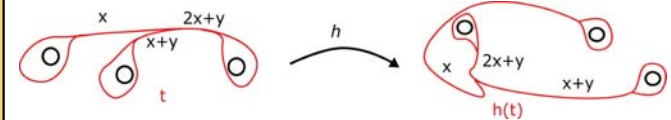
Representing curves as measured train tracks:



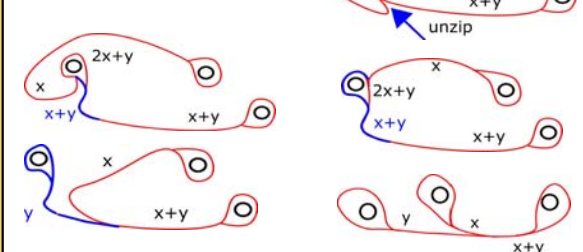
A basis of train tracks:



Image of a train track under a homeomorphism:



Unzip  $h(t)$  to obtain a basis train track:



**In progress work:** Generalize across all surfaces, homeomorphisms, and curves.