Bounds on Entropy in Subgroups of the Braid Group

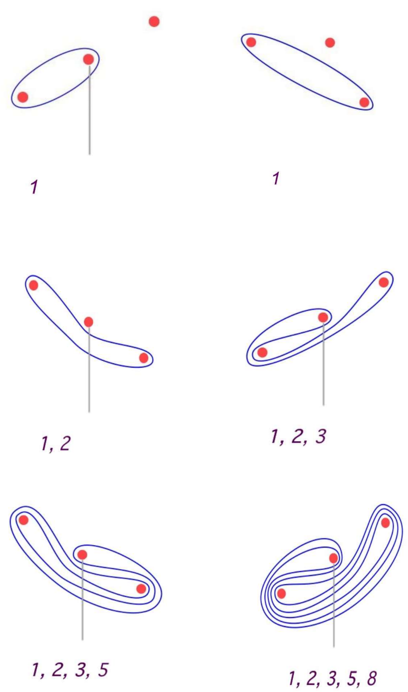
Georgia Institute of Technology

Sidhanth Raman, Ruoyi Wang, Nia Walton
Advisors: Dan Margalit, Hyunshik Shin

Introduction

Nielsen-Thurston:
Every braid has a number associated to it called its entropy, which is a characterization of how complicated the braid is.

Simple Example-Fibonacci Sequence



Entropy of this braid is the golden ratio!

The Problem

Results

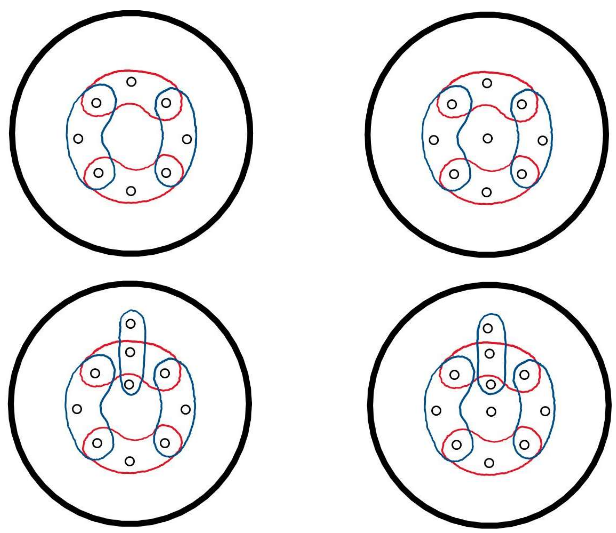
$$L(BI_n) \asymp 1$$

$$L(B_n[m]) \asymp 1$$

$$L([B_n, B_n]) \to 0 \quad \text{as} \quad n \to \infty$$

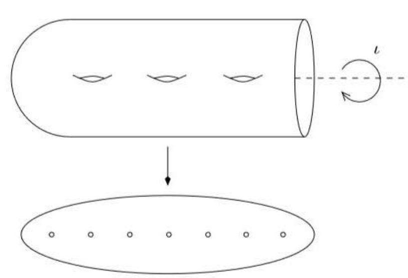
The Braid Torelli Group

$$BI_n = ker(B_n \to Sp_{2n}(\mathbb{Z}))$$



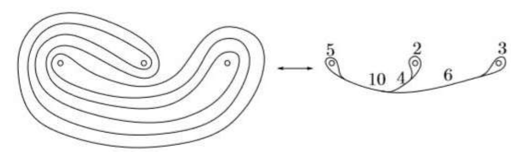
Level-m Congruence Subgroups of the Braid Group

$$BI_n \leq B_n[m] \leq \mathrm{Mod}(S_g^b[m])$$

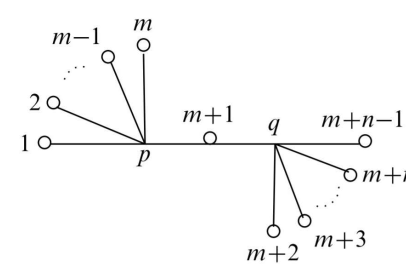


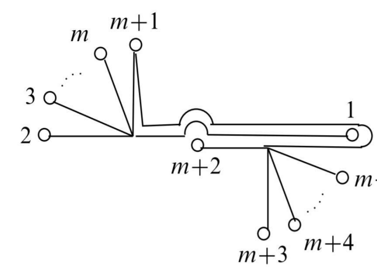
Methods

$$\text{Braids} \xleftrightarrow{\text{Train Tracks}} \text{Matrices}$$



Converting complicated curve to train track.





The corresponding train track for the braid in the commutator subgroup used to show our third result.

Acknowledgements

We would like to thank the Georgia Tech Mathematics REU for this opportunity, and Dr. Dan Margalit and Dr. Hyunshik Shin for their mentorship. This research is supported by the NSF.