Classifying Maps from the Braid and Symmetric Groups

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Totally Symmetric Sets

Goal: When are there NO interesting maps from the Braid and Symmetric groups to another group?

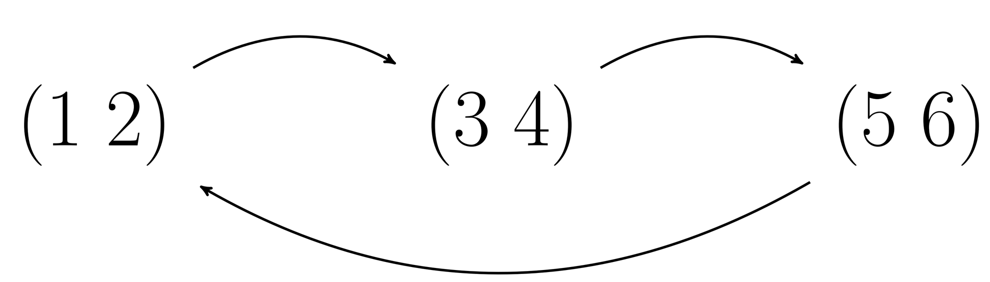
A Totally Symmetric Set is a subset of some group such that:

- All elements commute
- Any permutation of the subset can be achieved through conjugation by a group element.

TSS Condition: Under a group homomorphism the image of a totally symmetric set of G must be a totally symmetric set of the same size in H or a singleton.

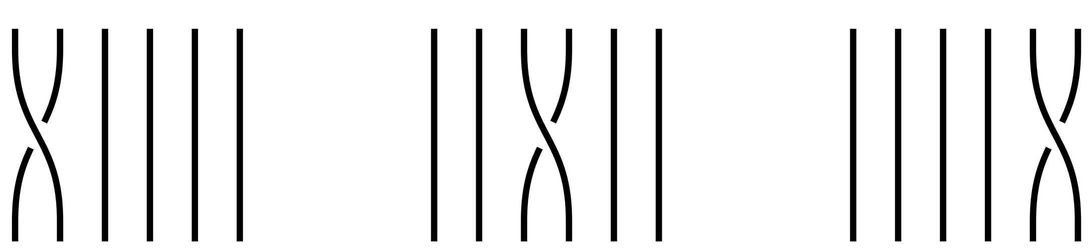
Examples

Symmetric Group:



Conjugating Element: $(135)(246)$

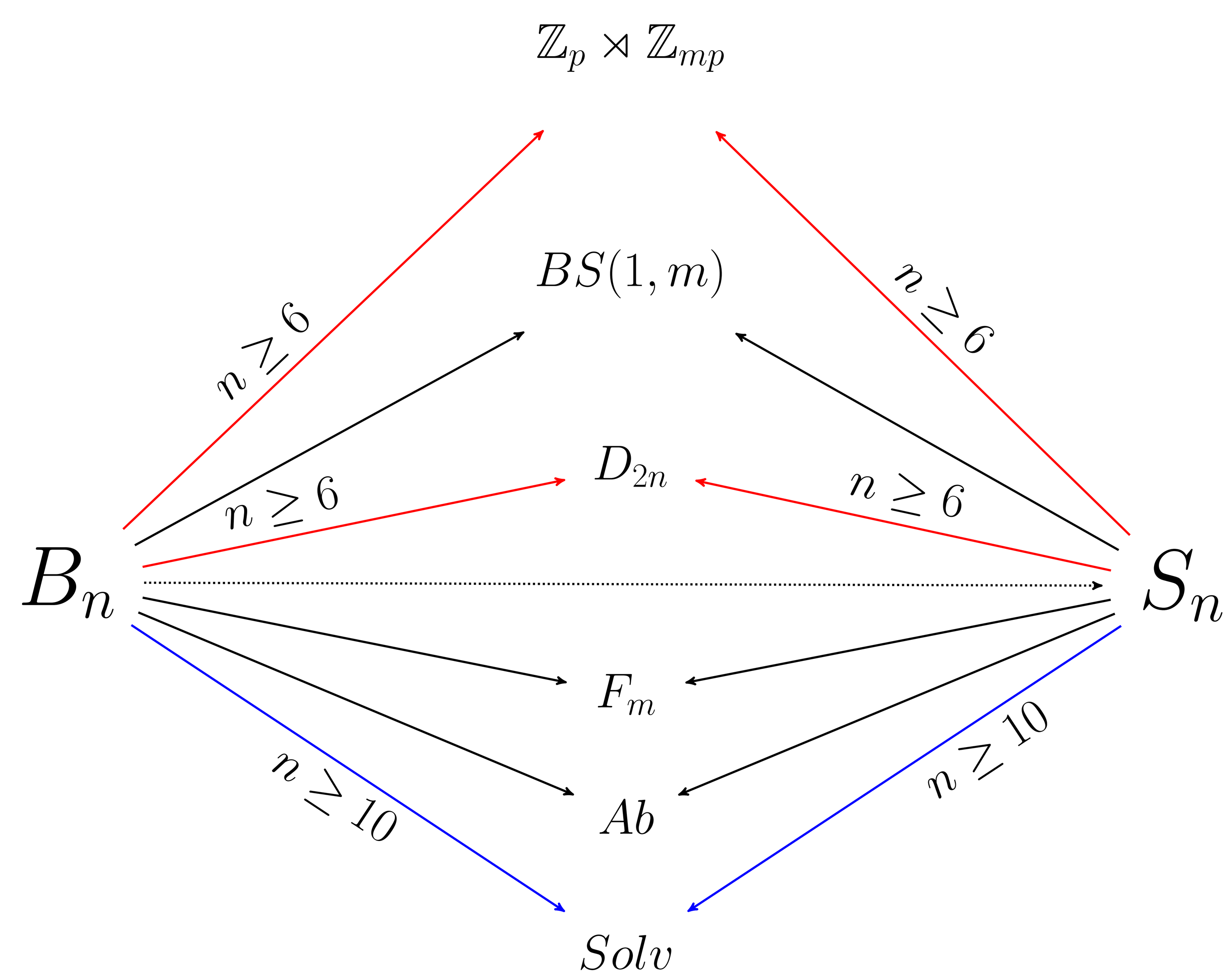
Braid Group:



σ_1 σ_3 σ_5

Main Results

Arrows ($A \to B$) signify that all homomorphisms from group A to group B are cyclic



Mapping Property

Proposition: If two elements of the totally symmetric set

$$\{\sigma_1, \sigma_3, \ldots\} \subset B_n$$

are sent to the same element, then the map is **cyclic**.

Proof. $\langle\langle \sigma_1 \sigma_3^{-1} \rangle\rangle \cong B_n'$ □

Sizes of Totally Symmetric Sets

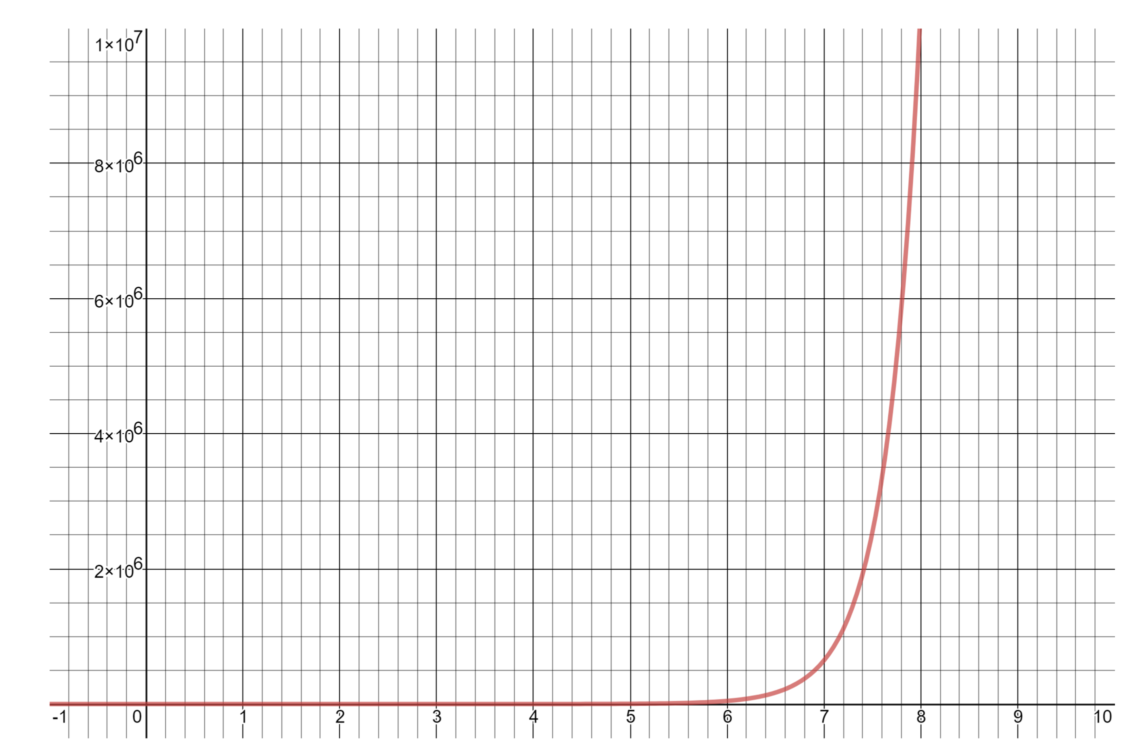
Let $S(G)$ be the size of the largest totally symmetric set in G:

G	$S(G)$
F_n	1
D_{2n}	2
$\mathbb{Z}/np \rtimes \mathbb{Z}/p$	2
$BS(1,n)$	1 or 2
$SL_2(\mathbb{C})$	2

G	$S(G)$
B_n	$\lfloor \frac{n}{2} \rfloor$
S_n	$\geq \lfloor \frac{n}{2} \rfloor$
$Aut(F_n)$	$\geq n$

G	$S(G)$
$G \times H$	$\max(S(G), S(H))$
Ab	1
Odd	1
$Solv$	≤ 4

Size Criterion



Let S be a size n totally symmetric set in G, such that no element in S can be expressed as a product of the others. Then $|G| \geq 2^n n!$

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