Counting Filling Pairs on Surfaces

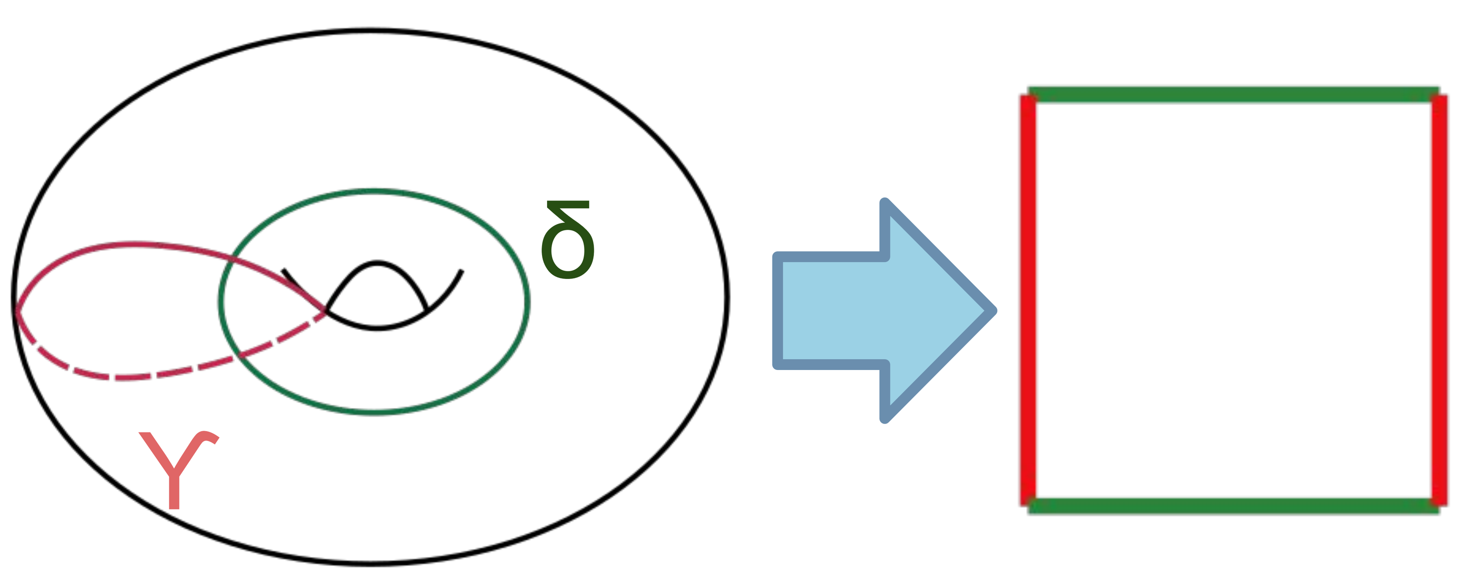
Ellie Cirillo[1] , Jiamin Li[2], Alice Ponte[3]

Mentors: Dr. Wade Bloomquist[3] and Dr. Dan Margalit[3]

Filling Curves on a Surface

- Two curves are **filling** if they cut the surface into a collection of disks.
- If a pair of filling curves intersects **minimally**, it cuts the surface into a *single* disk.

Filling Pair on the Torus

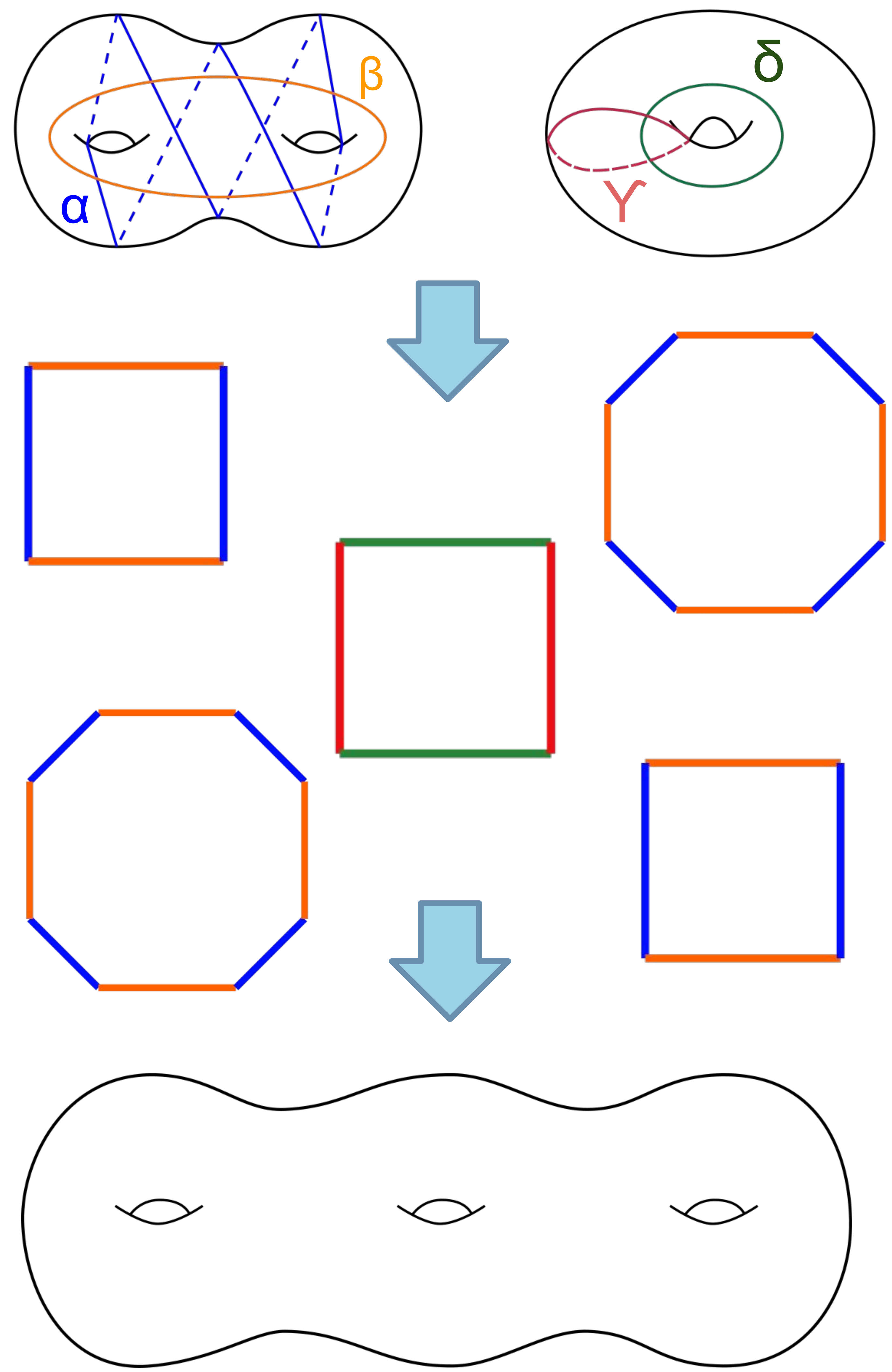


Main Question

How many distinct filling pairs of minimally intersecting curves are on a genus g surface?

$n(g)$ = # distinct filling pairs

Building Surfaces with Filling Pairs of Curves



Aougab and Huang Construction

Theorem

$n(3) = 12$ $n(4) = 672$

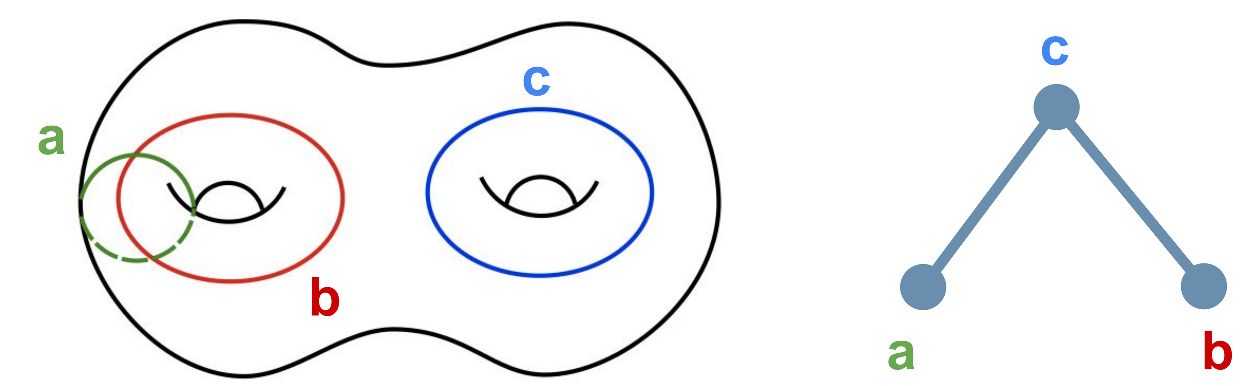
Technique: pairs of curves ~ permutations

Note: only 8 pairs are **decomposable.**

Upper Bound for Genus g Surface

$$2^{2g-2}(4g-5)(2g-3)! - 2(2g-1)\left[2\cdot 2^{2g-4}\cdot(2g-4)! + (2(2g-1)-6)^2\cdot 2^{2g-5}(2g-5)!\right]$$

Distance in the Curve Graph



Palaparthi - Mahanta (2021)

(a,b) filling pair → $d(a,T_b(a)) = 4$

Corollary

We have new examples of distance 4 curves.

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[1]*Christopher Newport University,* [2]*Agnes Scott College,* [3]*Georgia Institute of Technology*