A CRASH COURSE IN APPLYING TO A MATH PHD

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Abstract. A rough guide on what to do and expect when applying for a PhD in mathematics. Goes over broad topics before, during, and after the application process. Advice in this paper can still help people applying to PhDs in other fields as well.

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1. Introduction

Applying to grad school is no easy task, and it is very difficult to even know where to begin. This guide is specifically catered to current math undergraduates applying for a math/applied math/statistics PhD, but I am sure a good chunk of this will apply to other majors as well. I have learned a lot throughout my whole application process, and I wish I had some sort of general guide to follow while applying.

You absolutely do *not* need to follow everything I say in this guide. For example, research (section 3.3), graduate classes (section 3.4) and outreach (sections 4.6 and 5.2) are definitely not *needed* to get into a program, but I stress them heavily in this paper as they are all very strong ways to give your application a significant advantage. Also, I am saying what worked for me and what I found effective when I applied. There is a good chance this will not work as well for you, so adapt and ask others for guidance.

Throughout this crash course, a lot of the advice I give involves handling and speaking with others. You only have one brain; if you work with others you can have so many more brains working your application. Because of this, I am going to recommend you read the book *How to Win Friends and Influence People* by Dale Carnegie, which teaches you how to speak with people effectively and be a more sincere and affable person. This will help greatly in nearly all sections I wrote in this guide. You definitely don't need to read it, but it has helped me so much that I had to include it; *I do not think I could have gotten into grad school without the knowledge I gained from this book.* It is pretty popular and is most likely in your school's library.

Regardless, all recommendations given here are meant to be followed with sincerity and respect. **The key to success is genuine interest and enthusiasm for the field**; dishonesty will only hurt you in the long run. When you tell a professor you would like to work with them, say it because you mean it. **Don't lie to someone just to gain an advantage**. Collaboration is a big part of academia and everyone appreciates people who work with integrity.

I must say that I am not an international student, so this guide does not contain any pointers on things that are unique to international students, like applying for a visa or taking language proficiency tests (e.g. the TOEFL or Duolingo). Shintaro Fushida-Hardy's document *Grad School Application Advice* has some tips on applying as an international student (and he also has some other good resources on his website too!).

I hope you find value in this and it will help you achieve your goals. No matter the outcome of your application just remember that the fact you're reading this right now means you are already a motivated and driven student, aiming high and doing your best to achieve your goals. This will not go unnoticed, and it is important to stay motivated throughout this whole process.

2. Deciding if a PhD is right for you

Doing a PhD is *not* easy, it is not just "college 2". It takes tireless effort and will test your motivation and self-discipline more than your intelligence. You have to *love* math and will be neck-deep in it for a *long* time. I am saying this not to discourage you, but to inform you. I am not even trying to say this as if it's negative. If you love math and are willing to put in the work, then you have no

reason to worry.

With this being said, **do not think you are incapable of doing a PhD because you are not "smart" enough.** Doing a PhD never even entered my mind for my first three years of undergrad because I thought I was totally incapable. It took one of my professors to tell me with confidence *"I know you, and I know you are capable of getting accepted to a PhD program"* to convince me to give it a shot. I am so glad he told this to me, because this statement changed my life. Because of him I can say I have been accepted to my top choice, Vanderbilt University.

If you are considering a PhD, do not use your intelligence as a deciding factor. You do not know what you are capable of achieving until you go for it. Talk to your professors, friends, family, therapist about it and see if it is right for you. If you are still unsure, *just go for it.* If it's not for you, then you'll know it and you can move on. This is much better than thinking *"What if?"* for the rest of your life.

2.1. **You can apply for a PhD right out of undergrad.** Yes. This is not only possible, but common for math majors. I made this the title of this section because I had no clue this was even possible until a professor told me. In most other programs you almost certainly need a master's before getting accepted to a PhD program, but this is not the case in math.

In every other subject, this is the general track: after undergrad you get a job that relates to what you want to get your master's in. Then you get your master's, maybe work more, then do a PhD. In math (especially pure math) there are no jobs like this: you just do more math. Because of this, schools *want* you to apply right out of undergrad so you don't forget anything you've learned.

2.2. **What to expect.** If you get accepted to a PhD program, you will receive a master's in the first two years. This will require taking graduate level courses and passing qualifying exams[1]. *Qualifying exams*, or some schools call them *preliminary exams* (commonly referred to as 'quals' or 'prelims'), are exams that test a subfield of math, kind of like a final exam for all graduate students. Usually there are quals in algebra, geometry/topology, and real analysis, but it varies from school to school. These tests are obviously going to be very difficult, so to help study for them you are usually required to take classes in each of the qual topics. Usually the test is made in such a way that reflects what you learned in class, and the professor who taught the class writes or partially writes the qual.

The following three to four years will be primarily research focused, and at the end of it you receive your PhD. It typically takes 4-6 years to finish a PhD, usually 5. The type of research you do can vary a lot, so there is no way I can touch on it all. You will get research experience hopefully by your 2nd year, so you can see if this is something you enjoy doing.

2.3. **Mastering out.** If you have heard the term *mastering out*, it refers to the act of getting into a PhD program, finishing the requirements to get a master's, then dropping out before finishing the PhD requirements. Some people get into a program with the intention of mastering out, as PhD programs *pay you* to enroll, compared to master's programs which make *you pay*. Every department wants you to finish to the end, but they are also not clueless to the fact that people will try

[1]It's rare, but some schools don't even have quals, for example Brown University.

to master out.

I am not encouraging you or discouraging you to consider mastering out, just know that it is a thing. Schools do not like students who intend to master out, so don't even mention this in your application. Even then, not all schools allow you to master out, as some programs don't give you the master's unless you attain the PhD.

2.4. **Take care of yourself.** If you are thinking of applying a PhD, know that it is *very* important that you maintain your emotional and physical health during the whole process. This goes for once you're actually in a program too. Your health is more important than the school you attend, so learn how to manage your stress and take it very seriously.

I'm not an expert on mental health, so there is only so much advice I can give on this topic. Be honest with yourself and **seek help if you think you need it**. Don't be afraid to reach out to close friends/family as this can definitely be a time of great anxiety.

3. Before the application opens

So you've decided to apply for a PhD! Even if you're a first year undergrad and are few years away from applying, you can still begin preparing. This section will go over things you should know before applying and actions you can take to set yourself up for success.

3.1. **Professors.** Right off the bat, the single most important part of your whole application process is your professors. They will be your number one resource throughout your whole application journey if you utilize them. Your professors have been through this process before and most likely have already helped others who are exactly in your position. They have more information and better advice than I will ever have, and **if any of your professors provide information which contradicts what is in this guide, listen to them. You should trust them more than any other sources you find. When in doubt you should consult them with any questions you have.** I personally have had close to zero negative encounters with professors when I have reached out, especially when you are kind, respectful, and genuine. They were once in your position and almost certainly had huge help from their professors, so they understand how daunting this whole process can be.

If you do not already go to your professors' office hours, participate in class, talk to them after class, etc., start doing that right now! Just like you and I, they will be more likely to want to help someone who they know is kind and friendly. And sometime in the near future they might write one of your letters of recommendation. This will also be touched on in section 4.3, but your letter writers should be people who know you well as an individual, so it is important to establish a genuine connection early on so they can build a positive image of you that they will reflect in their letter.

Stick with the professors who are helpful and nice and ask them for advice frequently. But obviously do not bombard them with way too many questions. They are important people with busy lives, and do not take it personally if they do not respond quickly or at all. It never hurts to send a follow up email 3-5 days after with a quick *"Hey! Just following up on this email. Let me know if there*

is anything else you need from me." They will not be hurt as they understand they frequently miss emails, and if they are hurt then you should most likely stop connecting with this professor as they may not be too keen on helping you in the future (see section 4.3). Just read the room and treat them with respect: you'll be golden.

3.2. **Go to class.** You want to prove to the admissions committee that you are intelligent and capable of handling the workload of grad school. These programs pay *you* to enroll, so they do not want to waste their money on someone who can't complete their program. One of my professors who has been on numerous PhD committees told me that with each PhD applicant there is a risk factor, specifically a risk that they will drop out. They give acceptances to the applicants which have the least amount of perceived risk: you want to prove this to them, and first off you need to do well in your undergrad classes.

'If you can't perform well in your undergrad math classes, how could someone think you can do well in graduate math classes?'

This thought is not necessarily true, as anyone can have unexpected extenuating circumstances. But admissions will not be as kind. They have to sift through hundreds of applications, and would like a quick way to separate the applications into ones which are worth their time and ones that are not. It is very easy to toss applicants with low GPA's aside in favor of higher ones. I will not say what a "low" GPA is or give a "good" GPA you should aim for, as this depends on the caliber of school you are applying to and you should strive for as high a grade as possible in every class you take. But with that being said, if you have a GPA below a 3.0 it would be very difficult to get into a competitive PhD program unless you have some very convincing reason that you can explain in your application.

Of course, your math classes, especially your upper division math classes, are the most important parts of your transcript. I heard from a professor that your grade in real analysis is the single most important grade out of all your classes, but take this with a grain of salt as I have heard this from only one source. Regardless, make sure you hone in on these classes and do well in all of them. Of course, slip ups happen, but don't make it a repeated pattern. If they are actual slip ups, that will be reflected in your transcripts, and will be just a minor blip in your grades.

Try to understand the concepts of the classes as conceptually as you can, instead of just learning how to get an A in the class you are taking. What I mean by this is don't just learn how to do well on the final and call it a day: actually try to be *fluent* in the material. This will help you with your grades and in all your future classes, as there is often interconnectedness between classes. Of course there are times where you will not be as motivated to do this, because the professor is boring, or you have a harder class, or whatever. Be honest with yourself and take personal accountability for your own mistakes when studying.

Learn how to study effectively and see what works for *you*. Just because someone in your class who is really smart says that they like to wake up at 4:30AM and study for 6 hours straight does not mean that it will be effective for you. Experiment, read up on different techniques, and **make sure to take time out to rest and get away from math.** It is so easy to burn yourself out and having a healthy work-life balance, one where you can recover your mind and your energy, will be better for you in the short and long run. Resting can be anything from lifting, to reading, to sleeping, to partying: *just do anything that you enjoy that is not math.*

One of my professors recommends at least one day a week with no math at all, and although that may not always be possible try to strive for it when you can. Try not to over worry about your grades, yet simultaneously try not to beat yourself up about a grade you aren't happy with and overwork yourself. This is easier said than done, but keeping it in mind is the first step in making that your actual mindset.

If you are looking for a start on how to build a productive study routine and improve your self-discipline, I recommend at least looking into *Atomic Habits* by James Clear. It breaks down what you can do to build good habits and stop bad ones in a way that is simple and painless. The tips in there can be put to use immediately and are very effective.

3.3. **Research.** As was explained in section 3.2, you want to prove to the admissions committee that you are very capable of finishing a math PhD program. There are two main things that you will be doing in grad school: conducting research and taking grad classes. The latter will be covered in section 3.4.

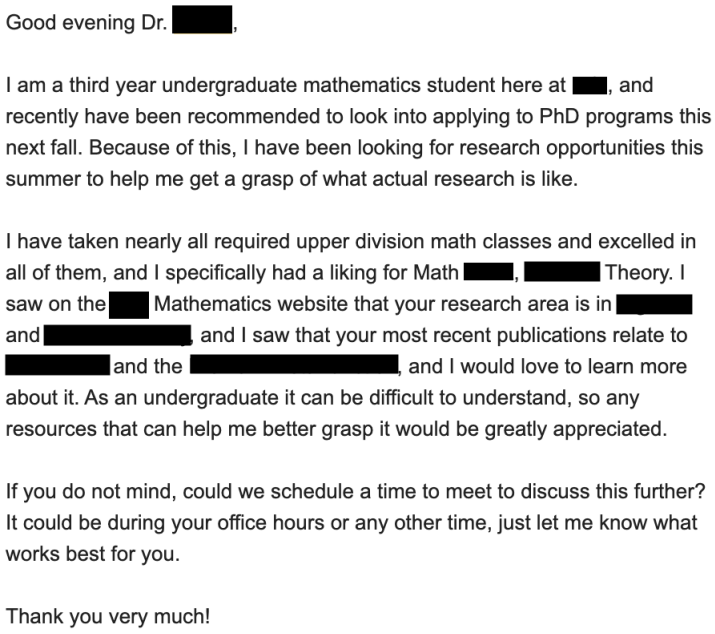
Your research experience can separate you from the crowd immediately, especially if you produce actual results, or better yet, get published as an undergraduate. However it can be very difficult to conduct actual research as an undergraduate in math, as you are still learning the fundamentals. Your professors are all researching in very niche subfields of math, and each requires a lot of background knowledge to understand.

If you're able to, **get involved in research as soon as you can.** You'll find research opportunities by reaching out to professors, explaining to them why you'd like to work with them, and proving to them that you are qualified. It can be very scary to cold-email professors you haven't spoken to before or haven't had any classes with, but in my experience they have all been nothing but positive and helpful. Figure 1 is a real email I sent to a professor I never met before. They responded saying they would be happy to meet with me in their office the following day. That meeting went very well and although I was not able to get a research position, it landed me a reading course[2] opportunity with them. This professor did not know me at all, but was still kind enough to give me a shot.

Sending effective emails is a very good skill to have, and should be something to hone in on. This will be very useful in communicating with your professors and reaching out to grad schools during the application process (see sections 4.6 and 5.2). Make sure to be clear and concise in your emails, and explain why you think you're capable of working with them.

It can be very easy to get lost when picking a professor to reach out to. Understanding their research by looking on their website is a good way to start, but it will likely get you scratching your head at exactly what their arXiv papers are saying. Try to find professors who do research you like, but definitely prioritize picking a professor who you think is nice over one who does research in your specific desired subfield of math. They will likely write you a letter of recommendation and it is important that they like you. This letter of recommendation will be more valuable than getting research in your specific desired area of research (more about letters of recommendation in section 4.3). As long as you are mildly interested in the topic and you put in the work, the exact research project doesn't really matter. If you can, reach out to professors who you know are nice. Some professors also put on

[2]Definition of reading course is on the next page.



Good evening Dr. [redacted],

I am a third year undergraduate mathematics student here at [redacted], and recently have been recommended to look into applying to PhD programs this next fall. Because of this, I have been looking for research opportunities this summer to help me get a grasp of what actual research is like.

I have taken nearly all required upper division math classes and excelled in all of them, and I specifically had a liking for Math [redacted], [redacted] Theory. I saw on the [redacted] Mathematics website that your research area is in [redacted] and [redacted], and I saw that your most recent publications relate to [redacted] and the [redacted], and I would love to learn more about it. As an undergraduate it can be difficult to understand, so any resources that can help me better grasp it would be greatly appreciated.

If you do not mind, could we schedule a time to meet to discuss this further? It could be during your office hours or any other time, just let me know what works best for you.

Thank you very much!

FIGURE 1. Cold-email I sent to a professor who didn't know me.

their website that they like getting undergrads involved in research so reach out to them too. Talk to other undergrads and ask them who they'd recommend for you.

Outside of conducting research in your university, you can also apply to REUs. *Research Experiences for Undergraduates* (REU) are summer programs which allow undergraduates to conduct research. You can apply to any REU you want, including out-of-state universities. I have no personal experience with REUs, but I know they can be competitive to get into. REUs are a very popular option, so I would recommend reaching out to your friends and professors about it.

If you can't land a research position, don't worry. There is still something you can do. You can ask professors if you can do a reading course with them. A *reading course* (also known as *directed reading* or *independent reading*) is when you read a graduate level textbook (take notes on it, solve its example problems, etc.) under the guidance of a professor or graduate student. You typically meet with them weekly or biweekly and can ask them questions about anything you read. Reading courses are a great alternative to research, and there are usually plenty of opportunities if you look for them. In my experience, professors who are not able to include you in their research can offer you a reading course.

You can also get units for doing reading courses, which is a great way to get yourself up to full-time status as a student. This can be very useful if you rely on financial aid, as they typically have a lighter workload compared to a normal class.

3.4. **Graduate classes.** As explained in section 3.3, grad classes are the other half of minimizing your applications perceived "risk". Again, you do not need take any grad classes to get into a program, but taking them can only improve your application, and will likely do so drastically. The specific class does not matter too

much, but for your own sake it'd be best to take graduate real analysis, algebra, or topology. All grad schools I have seen have those classes as requirements to receive a PhD, and familiarizing yourself with any of these topics nice and early will accelerate you once you are actually in grad school.

Go ahead and pick your favorite class, you have some freedom here. In my own personal opinion, picking a great professor is more important than picking a topic you're more interested in. In my experience and in experiences of those I have spoken with, great professors can make a bad topic enjoyable, but a bad professor can make anything unbearable. Look at their Rate My Professors page and read the reviews. Notice any general trends in what people say. Talk to friends who have had that professor's classes and ask about their experience. If you realize the professor is bad, you can transfer to a different grad class with a better professor as long as the class enrollment window is still open.

You will most likely need an authorization code to enroll in the class, so email the professor who is teaching the class and ask them what steps you need to take to enroll.

Grad classes can take a lot of time. I do not personally recommend stacking multiple grad classes at once since it can potentially affect your grades in them, and worse, your actual undergrad classes (see section 3.2). It is much better to do very well in a few classes than to do just okay in a large number of classes. In the end it is your call: do what you think will work best for you and be honest with yourself on what you can handle.

3.5. **GRE.** The *Graduate Record Examinations* (GRE) exam is a standardized test meant for graduate students (think the SAT for grad school). There are two GREs you need to think about: the general GRE, and the math subject test GRE. The general GRE is very similar to taking the SAT, as it covers reading comprehension, vocabulary, and basic math. The math GRE is easily much more important, as it covers topics in college level math. It tests single and multi-variable calculus, real and complex analysis, group and ring theory, topology, and probably more but those are the main topics. The order I just gave is in rough order of what is most prevalent in the test.

For both GREs, schools either require it, leave it as optional, or do not even give you a space to submit it. Figure 2 shows a table of the schools I actually applied to and their requirements for the general/math GRE. From this you can see that the math GRE is more important than the general, and the general is often not even considered. It seems that more and more schools are omitting the requirement, but there are schools that do require both GREs, for example UC Santa Barbara.

You have the decision of whether or not you would like to take either of the GREs. It definitely depends on where you want to go and their individual requirements. You can take the general GRE if you want/need to, but no matter what I definitely recommend you take the math GRE. It opens up the doors to apply to most schools and it will show that you put in some serious dedication to your application.

That being said, make sure to study early for math GRE. The test is *hard*. It is 66 multiple choice questions that you have to do in 2 hours and 50 minutes, giving an approximate average of 2.5 minutes per question. It is very easy to go too quickly and make a mistake, or get stuck on a problem and waste more time than you realize.

I cannot give too much advice on how to do well on the test as it just comes

School	General GRE	Math GRE
1	Not accepted	Not accepted
2	Not Required	Optional
3	Not accepted	Optional
4	"Not considered"	"Not considered"
5	Optional	Optional
6	Optional	Optional
7	Not required	"Highly recommended"
8	Not Required	"Strongly recommended"
9	Optional	Required
10	Not Required	Required
11	Not accepted	"Strongly recommended"
12	Optional	Optional
13	Not accepted	Not accepted
14	Not Required	Not Required
15	"Not required or considered"	"Not required or considered"

FIGURE 2. GRE requirements of the schools I applied to.

down to studying for it and taking practice exams. There are resources out there online and books that can help you study. You can click here for a forum post I found that gives some helpful advice on how to study for the test, although it was written in 2018. Some things on there might be outdated, which is why I didn't write them here.

I recommend studying as early as you possibly can. Realistically, begin studying around January or February when you are beginning winter of your third year. The math GRE is only available in April, September, and October: if you begin studying in January you have four months until you take it in April, and from there you have five months to study further and take it in September. Definitely try to take it as many times as you can as these tests are random in difficulty.

When receiving your score back, the raw number of questions you got correct does not matter as much as your percentile score. The percentile pretty much says how well you did compared to everyone else who took that specific test. Since the math GRE is very difficult, it's tough to get a good score. Determining what is a "good" GRE score varies a lot from who I have asked. I heard professors say it varies from school to school, such as the top 10 schools only want 90th percentile or higher. I also heard to submit it as long as you are above the 50th percentile, since by definition you are above average. A lot of current grad students I have asked were in the 60-70th percentile range. Ask around, make your best judgement.

Mentally, do your best to keep a healthy mindset throughout this. It can be very draining and studying for the GRE led me to doubt if I even wanted to do a PhD anymore. The first time I did the math GRE I got in the 30th percentile after studying *hard* for a whole summer.

The good news is that many schools seem to have it as optional, so all hope is

not lost if you get a poor score: just don't submit it. As a word of caution when signing up for the math GRE, **do not choose to send your scores to schools before taking the exam**. Yes, it allows you to send your scores for free, but the catch is you cannot unsend your scores once you have taken the test. This led me to send my 30th percentile score to my top four programs and it absolutely broke me. Yes, you may have to pay more later on, but I personally would much rather pay more if it secures the fact that they do not see a very bad score.

3.6. **CV.** A *Curriculum Vitae* (CV) is a resume of your academics. It consists of your name and contact info, the schools you have attended, GPA, research, presentations, awards, and applicable skills (e.g. languages you know, programming knowledge). You can also include upper division math classes you have taken, positions as a TA/LA, or anything else you think could be relevant. Look at the CVs of your professors, grad students, and see what theirs look like. From there find a template that works for you and begin writing. There is definitely a lot of freedom in how you form your CV, but I will give advice I believe made my CV stand out.

Do not add unnecessary filler into your CV. For example, if you did a mini-project as a final for a regular class, don't add that. Admissions can sniff it out and see that it is only in there to fluff up your CV. Only include things that make you unique as an applicant. Unless you are an unbelievable student with loads of legitimate research, your CV does not need to be more than two pages.

Also try to format your CV in a way where the most important things about you are at the top, think like a tier list. This is not a strict guide, as most people always put their education up top, but use it to help you stencil out your first few drafts.

If you can, ask your professors their opinions on your CV and what you can change/improve. Listen to them as they know exactly what they're talking about. Make your CV look nice and make sure there are no errors. You would not want an admissions officer to catch a misspelling on your CV. Don't leave it to the last minute. Reread it multiple times.

You can write your CV in a standard Google Doc or in LaTeX. Choosing one comes down to personal preference; in my opinion LaTeX looks cleaner and more professional. I think putting in the extra work can go a long way. Click here for some Overleaf CV templates, and click here for the specific template I used.

3.7. **NSF.** The *National Science Foundation's Graduate Research Fellowship Program* (NSF GRFP) is a fellowship which you can apply to for additional funding during your time as a graduate student. Receiving this can be a career-changing difference, as it pretty much buys you out of TA-ing for a good chunk of your time in grad school.

You can apply to the NSF fellowship as a last year undergrad, and I have heard that it is more likely to get it as an undergrad compared to as a grad student (not sure how true that is, but it is something that has been talked about). The application requires letters of recommendation and a proposal of what you plan on researching in grad school. You do not actually have to follow through with your research plan, as once you earn it they will not take it away from you.

Getting NSF funding can potentially get you into programs you have been rejected from. For example, if you get rejected from everywhere you'd like to go yet

receive NSF fellowship, you can email the departments saying that you have NSF. Now that the department no longer needs to pay for you, it is possible that they decide to admit you. I know people who have done this and been admitted.

Before enrolling in any schools, definitely receive guidance from professors and others and make sure you are still able to afford grad school once the grant runs out. You can click here for a very comprehensive guide on how to apply to the GRFP. My friends refer to the primer as the "bible" for applying to the GRFP and has been extremely helpful for them.

3.8. **Extracurriculars.** Aside from all this, there are always other little things you can do to boost your application. To list a few: being a TA/LA, being involved in a math/STEM related club on campus, working in a math/STEM related job, etc. All of these help your application but are certainly not required. Keep these things in mind and only do them if you would normally. I heard from undergraduate admissions that they can usually sniff out who truly cares about their extracurriculars and who just does it to include it on their application. Make sure to legitimately enjoy them and writing about them in your essays will be significantly easier.

4. Beginning the application

The applications are opening! This will start around September-October, and they will stay open until about December-January. Each school has a different start and end date for their applications, so make a spreadsheet of all of them to keep yourself on track. I recommend opening up all the applications and making their portal accounts as soon as it opens. Include the website to their application portal on your spreadsheet, it just makes it easier for you to access later.

4.1. **Deciding where to apply.** You would think this is the easiest part but it becomes very complex very quickly. We all want to apply to the Ivy Leagues, UCLA, Stanford, and all the places with high prestige, but if we are being realistic we cannot solely apply to these schools. We need to have a good spread of safety, target, and reach schools. And we need to make sure to pick schools that have good programs for what we are planning on researching.

It can be surprisingly difficult to find which departments have what strengths. There is no list anywhere that mentions this specifically, and in my experience this knowledge is primarily passed on through word of mouth. For example, applied and computational math at Dartmouth College has about ten professors, while their logic department has only one. It would be a great idea to apply there if you are in applied math, but not ideal if you are in logic.

Looking through *every* research paper of *every* professor at *every* university you are interested in is exhausting. So make it easier on yourself and talk to your professors and TAs. You should still do your own research on which schools to apply to, but utilize them to make your life easier.

'But how many do I apply to?' I have heard a wide range of answers. I know people who have applied to 40+ schools and I know people who applied to 1. The average answer I have heard is about 12-18, with people on the higher end typically admitting how they wish they applied to fewer programs. I think this is a good range and I think applying to more than 18 seems overkill, and any less than 10 feels a little risky. Look into a bunch of schools and try to have an even spread of safety, target, and reach schools. For me, I applied to 15 schools, with approximately 6

School	Difficulty (Internet)	Difficulty (Prof. 1)	Difficulty (Prof. 2)
1	Giga reach	Reach	Not happening
2	Giga reach	Reach	Not happening
3	Giga reach	Reach	Not happening
4	Reach	Reach	Not happening
5	Reach	Reach	Reach
6	Reach	Target	Reach
7	Reach	Target	Reach
8	Reach	Target/Safety	Target
9	Reach	Safety	Target
10	Reach	Safety	Target
11	Target	Safety	Target
12	Target/Safety	Safety	Target
13	Target/Safety	Safety	Target/Safety
14	Safety	Safety	Safety
15	Safety	Safety	Safety

FIGURE 3. Difficulties of the schools I applied to according to different sources.

reach, 6 target, and 3 safeties.

'But how can I tell if a school is a reach, target, or safety?' Very good question with no good answer. This all depends on who you ask and the strength of your application. The best thing you can do is ask professors (hopefully ones who have been on admissions committees), especially your letter writers since they will know you and your application the best.

Figure 3 shows all 15 schools I applied to, with the first column based off of results I saw on the internet, and the other two based off of two different professors that I asked. For the first column, I primarily used mathematicsgre.com, which is a website where people talk about PhD applications, and, more importantly to us, report their PhD decision results alongside their application stats. **(Disclaimer: It is very easy to become obsessed with looking at websites like these. People on these websites have unbelievable stats and will make you feel like your application is worse than it actually is. This will make you feel bad about yourself. Be wary and don't sink too much of your time or emotions into them)**. As you can see, Professor 1 was significantly more confident in me compared to Professor 2, so I recommend asking as many professors as you can to get the most balanced take.

I also wanted to mention that every professor I have spoken with says that it is not ideal to apply to your undergrad school for a PhD. You want to broaden your horizons, and sticking to the same department will limit yourself.

4.2. **Personal statement essays.** Each school has different requirements for their essays. Some schools have two essays, some schools have only one. Here are the two categories of essays I encountered:

(1) Academic Statement: This asks about your academic history, research experience, research interests, future plans, motivation for going to grad school, and why you specifically want to apply to their program.

(2) Diversity Statement: This asks you how you will contribute to diversity at their campus and promote inclusivity by explaining your own personal life experiences.

Every program will ask for an academic statement, and only some will ask for a diversity statement; from my schools about half of them asked for a diversity statement.

The academic statement is easily the most important of the two. Try to hit all the points I outlined in (1). Here are some tips when writing:

(1) Start early: Begin drafting out and writing your essay as soon as you get the prompts. Do not wait; the deadlines tend to interfere with finals and the last thing you want is to be double-booked during that time. It will make your grades, essay, and life better if you start early.
(2) Seek advice: Go to your school's writing center and ask for help. Ask your professors to read it over and give feedback. Ask what admissions usually look for, and if they have ever been a part of an admissions committee.
(3) Make a template essay and alter it to each specific school: I wrote one essay, and left spots blank so I can fill in specifics later. Leave spaces for which professor you want to work with, why you specifically want to go to their school, etc. You can always shrink it as needed for schools that have different page/word limits.
(4) Call out a professor you want to work with by name, but only one: I heard that calling out multiple professors in your essay comes off as being desperate, as if you'll do anything to get into their school. Find one specific person, look into them, and personalize that portion of your essay to them. Do they have a big working group you want to be a part of? Have you already done research in their area? Have you read and enjoyed one of their papers/books? *Have you already spoken with them before?* All great things to mention in this section. There is a chance that your essay gets passed along to that professor, so make a good first impression.
(5) Be concise and readable: The committee *will* skim your essays. They have to read hundreds of them. They will appreciate a straight and to the point essay. Read your sentences out loud and see if they flow off the tongue. If there are any sentences that are hard to read it will disengage them from your essay, so avoid filler words/sentences/paragraphs.
(6) Bold what you want to stick out: Notice how **this** catches your eye before *this* does? Since the committee will skim your essay, bolding will quickly show them what's important. This advice came from a professor who was on a committee himself, who said that he appreciates when essays include bolded words since it makes it easier to discern what they should be paying attention to. Things to bold are your research experience, grad courses, unique honors/awards (not just Dean's honor list), which specific professor you want to work with, and anything else that makes you stand out. For example, I had plenty of on-site teaching experience in elementary, middle, and high schools through the CalTeach program. My professors told me to lean into this a lot, since this is very rare to have in an applicant and will set me apart greatly: I was told to bold the whole sentence where I describe that I have 100+ hours of student teaching experience. But make sure you don't bold too much, only do it if something is really significant.

(7) Do not begin with "Ever since I was little...": My professors have told me that this is the biggest cliché that nearly *every* essay tends to have. They don't care that you always liked Rubik's cubes and this somehow made you like algorithms: they just want to know if you're worth the money they might spend on you. That being said, it is very difficult to start an essay without saying something along these lines, especially when the essay asks what motivated you to do a PhD in math. Maybe you don't have a specific reason: you just like math. No problem, you don't need one. You are not proving why you are passionate about *math*, but why you are motivated to do a *PhD*. For example, I wrote that my main motivation was my parents, as I am a first generation college student and getting a PhD would change my family's lives. I also wrote that I liked teaching and looked forward to being a TA. This has nothing to do with math at all, and those sections of the essay could have been written for any PhD, whether it's math or chemistry.

(8) Don't be afraid to include some actual math: In my application, I had actual research with results. I was recommended to explain the project in my essay, but I found it difficult to without mathematical symbols. My professors' solution was to write my essay in Overleaf and include the raw math. For example, it is more clunky to write "for a twice continuously differentiable function, we know that it is also once continuously differentiable" compared to "for $f \in C^2$, we know $f \in C^1$". The people reading it are math professors who will know what you're talking about, so if it helps your essay, add it in. Most schools have a PDF upload for your essay, but some have a text box which forbids non-English characters. When this happened to me I emailed whoever was in charge of admissions and asked if they could accept my PDF instead; they accommodated me every time.

I know this is a lot to take in, so take your time and read it over more than once. And feel free to bend these rules a bit if you need to. As a reference, here is an excerpt from my actual academic statement. It explains the results from *Generalized Bounded Distortion Property*, a paper I co-wrote in undergrad.

> That summer I was involved in **research in dynamical systems** under Professor Anton Gorodetski, where I worked closely with his doctoral student Grigorii Monakov on our paper *Generalized Bounded Distortion Property*. The Bounded Distortion Property is well known in the theory of dynamical systems, most notably for its application in Denjoy's Theorem. It considers applying the same C^2 map n times on intervals in $\mathbb{R}$, and we generalized the theorem by proving it for a sequence of different $C^{1+\varepsilon}$ maps applied to curves in $\mathbb{R}^d$. The original lemma required two constants, C and L, where for twice continuously differentiable $f : \mathbb{R} \to \mathbb{R}$ and for $I \subset \mathbb{R}$, we have
>
> $$\frac{|f''(x)|}{|f'(x)|} < C, \qquad \sum_{j=0}^{n-1} |f^j(I)| < L,$$
>
> for all $x \in \mathbb{R}$. Our extension requires considering a new constant α which depends on the maximal angle of the curves, defined for a regular C^1 curve $\gamma : [a, b] \subset \mathbb{R} \to \mathbb{R}^d$ as

> $$\alpha(\gamma) = \sup_{x,y\in[a,b]} \angle(\gamma'(x), \gamma'(y)),$$
>
> and requires redefining C and L for composition of $C^{1+\varepsilon}$ maps
>
> $$f_1, \ldots, f_n : \mathbb{R}^d \to \mathbb{R}^d$$
>
> on a curve in $\mathbb{R}^d$. The result has potential to be applied to the study of spectral properties of discrete Schrödinger operators. The original Bounded Distortion Property has been applied to study the spectrum of the Fibonacci Hamiltonian, but our generalization allows for the substitution of other Sturmian sequences. **We submitted this work for publication**, and a preprint is available at arXiv:2309.06630.

Here is another excerpt, which was the stencil I used when talking about which professor I wanted to work with. I never followed this to a tee and I always personalized it with each professor.

> With that being said, I think it would be great if I could work closely with **[professor name]**. Their paper [paper name] is really interesting for [these reasons]. The program at [school name] would be a great environment for me to work in for [these reasons]. [Any extra sentences about why I like this school/this professor].

Here is an excerpt of my last paragraph, which detailed my motivation to pursue a PhD.

> Aside from my innate desire to learn and teach, my biggest motivation to pursue a PhD is my family. Both of my parents are immigrants who came to the U.S. as a way to better the future of their family. As a first generation college student, just finishing my bachelor's would be a massive milestone, but obtaining a PhD would be monumental. It would open up opportunities for me that no one else in my family has ever had. My parents worked far too hard to get into this country and get to the state that they are for me to pass up an opportunity as important as this, and I will work on this degree as if my family depends on it.

As for the diversity statement, the requirements fluctuate a lot from school to school, and what you write about is really personalized to you. I think schools are starting to require this type of personal statement to appear more equity focused to the general public, and I do not know how important this essay actually is. Just do your best to write something that fits what they are looking for. If you were involved in any clubs, teaching, etc. then mention it here. If it helps, here is an excerpt from mine.

> Crystal Cove Conservancy is a nonprofit organization that educates students about environmental research by letting them collect scientific data at Crystal Cove State Beach. They partnered with CalTeach and tasked us to revise their curriculum; so we went to the beach and participated in their program to understand it from a student's perspective.
>
> We learned there was a house along the coast that had a seawall, which can cause irregular sand levels of the beaches nearby it. Seawalls of this type are no longer built, but this one is still up because the house is a historic monument. We all wondered why it is still there if it hurts the beach, but we realized there

is not a simple answer to this.

We decided to use our curiosity as a source of inspiration for our lesson. We created a four day lesson that addresses the question: "Should this beach house be removed?" It assigns students into small groups of different roles—such as "biologist", "historian", or "local resident"—and gives them two days to prepare for a mock town hall meeting by taking a position on the topic. During the town hall, the groups debate their viewpoints using data they collected themselves, whether it be from the beach or from any external resources they use.

Our lesson motivates students to learn more about real environmental issues in a unique way that is memorable. It promotes sharing ideas with colleagues and having an open mind about opinions different from their own. We also made sure that three of the four days of the lesson occurred in class, as we understand not every student has the economic or physical ability to participate in the activities at the beach.

4.3. **Letters of recommendation.** Surprisingly, I have heard from many professors that your letters of recommendation are more important than your personal statement essays. It's to do with the fact that your letter writers should be well respected in their field, so their word is "worth more" than yours. When choosing your letter writers, you want to make sure that they know you well as an individual. Hopefully you have been following my advice from section 3.1 and you have already made an impression on your professors.

When deciding who to ask, you want to prioritize people who can back up important parts of your application, so definitely ask your research professor or professor from a grad course you took if these apply to you. You ideally would want them to be active in math research, and the "bigger" they are in math (i.e. how good their research is) the better. This is the unfortunate truth when it comes to letters of recommendation, as even though a lecturer may know you very well, a professor with high research activity will be taken more seriously. But do not let this be your only deciding factor; it is far more important to pick a letter writer who you connect with genuinely than to cherry-pick professors with "high" status.

Every school I've seen requires at least three letters of recommendation. Some only allow three, while some allow up to five. Ask your professors to write you a letter of rec as soon as you can (September/October). You do not want your letter writers to be the reason why you do not turn in your application in on time, so get it done early. Professors need time to write your letter, and may ask for a 2-6 week notice beforehand. Ask them this in person if possible. This may seem a little strange, but I have a good reason. This is a very important piece of advice I will tell you right now: **only get professors who are enthusiastic about you to write you a letter of recommendation.**

I have heard many real stories of professors writing negative letters for their students. I have heard about a professor writing a letter which is a sentence long, only saying "He did well in my class". I even heard of an essay which outright said that this applicant is unfit for research and that whoever is reading this letter should not admit this student. *You want to avoid this at all costs*, and is the reason why I said to only associate with professors who you like and like you (section 3.1). Asking them in person will be a much better way to judge how excited they are about you.

Some of my professors recommend asking your letter writers if they think they can write you a "strong" letter. Any professor who knows me can write a half-decent letter, but only the ones who know me well can write me a *strong* letter. In my experience these professors know what a "strong" letter means, and will (hopefully) tell you the truth.

Now that you have enough people who agreed to write you a letter, you want to send them all the information they need and make it as smooth as possible. Their inboxes are flooded all the time, so you want to send them one nice email which contains all the information they need. I used this page by Professor Joseph Silverman of Brown University as a guide. Here's a summary of what to include:

(1) List of schools: Make sure to email them a new list if the list changes.
(2) List of classes you have taken with this professor
(3) Who else is writing you a letter
(4) Specific information you want them to include in your letter: If you think you were one of the top people in their class and showed up to their office hours frequently, remind them here to mention that.
(5) Deadline: Instead of sending professors each individual deadline for when they send their letters, just send them the earliest deadline. This is less confusing and more convenient. They usually try to bang them out and submit them all in one sitting. I also personally like to tell them the deadline is 2-3 days sooner than it actually is, to give myself more cushion and less worry.

The professors I sent this format to have appreciated it a lot, and said it was the most helpful information they have ever received about a letter of recommendation from a student.

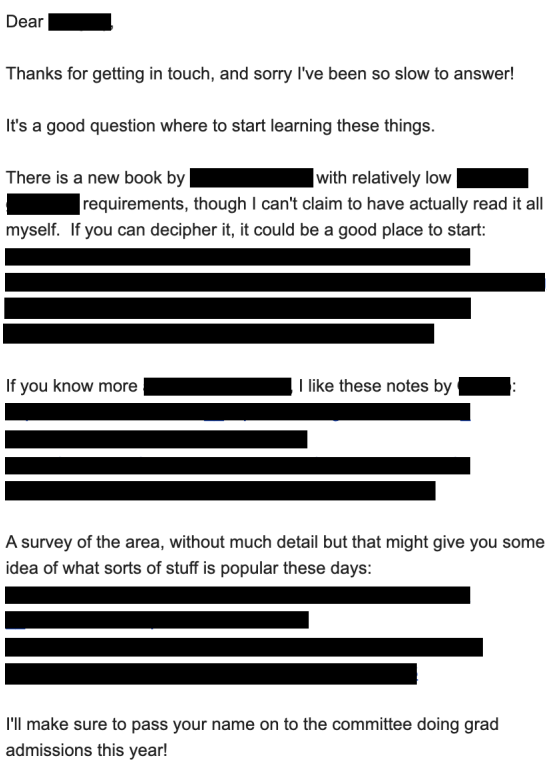
When you add your letter writers' email into your application portal, they will immediately receive an email on how to submit their letter. Do this all in one go for all your schools. Professors prefer this, because gives them a cleaner inbox compared to getting a dozen emails sprinkled throughout several days. It will also make them less likely to miss a school when they actually submit their letter.

4.4. **Fee waivers.** You have to pay to apply to pretty much[3] any school, but nearly all of them have fee waivers. An application can cost anywhere from $60-135, and this can add up quickly when you have 15+ applications. If a school grants you a fee waiver, you can apply there for free.

There are numerous ways to qualify for one, but the most common ones I've seen are to be a first generation student, to show that you have received financial aid in undergrad, or they make you write a short a paragraph on your previous financial struggles. Qualifying will vary a lot from person to person since everyone is unique.

4.5. **Transcript.** Very minor thing to mention here. All schools want you to submit an unofficial transcript, so there is no need to order an official one. Some schools may ask you to submit a transcript with the name of your actual university on it. When I went to my online unofficial transcript it didn't have it, so what you can do is ask for an official transcript in person at your school and just scan it to a PDF.

[3]Some schools are totally free! For example, Dartmouth's math PhD program has no application fee.


Dear [redacted],

Thanks for getting in touch, and sorry I've been so slow to answer!

It's a good question where to start learning these things.

There is a new book by [redacted] with relatively low [redacted] requirements, though I can't claim to have actually read it all myself. If you can decipher it, it could be a good place to start:

[redacted]

If you know more [redacted], I like these notes by [redacted]:

[redacted]

A survey of the area, without much detail but that might give you some idea of what sorts of stuff is popular these days:

[redacted]

I'll make sure to pass your name on to the committee doing grad admissions this year!

FIGURE 4. Positive response from a cold-email.

4.6. **Outreach.** If you play it right, this can be your golden ticket into grad school. As unfortunate as it is, it is not always the case that the most qualified applicants get admitted. Humans are the ones looking at your application, and humans are biased. You want to get in the admission committee's good graces, so you want them to know how great of an applicant you are.

There are so many ways to get your name into the minds of the admissions committee, and nearly all of them require cold-emailing faculty. This can be a lot of work, so I recommend doing this to the programs you want to go to the most.

You would be surprised how easy it is to reach out to the professors. Just tell them you're a student who's interested in working with them! Mention why you like their field of research, and ask what you can do to learn more about it. Keep it short and sweet. Be honest and be yourself; only reach out if you truly mean what you say. Professors can usually tell if someone is not genuinely interested in their subfield and disregard them. If they don't respond in a few days or so, follow up kindly. If they don't respond again then just leave it. If it goes well, you can try to schedule a Zoom meeting with them.

Figure 4 is a response I got when I cold-emailed a professor. I mentioned that I am applying to their school, and I asked them for resources I could use to better understand their research, as it can be difficult to dive headfirst into new math. Look at the response! I didn't even mention anything about it, but they said they passed my name on to the committee! It is unexpected things like this

that convinced me of the power of outreach.

Another idea is to reach out to grad students. This is not necessarily to try to get closer to the committee, but for questions about what it is like to live and be a grad student at their institution. They tend to be less busy and are very understanding if you ask some general questions about their university. I have scheduled Zoom meetings with grad students and have had some very insightful conversations. You can also try to reach out to current or past advisees of the professor you'd like to work with, and ask them what it's like being their student.

If any of your professors mention they have friends in other departments and recommend you apply there because of it, you should, no, *need* to reach out. This could be the sole reason of why you get into a grad school. It might feel cheap to have an "in", but this is the game that others are playing. Take all the opportunities you can get and email them saying you know their professor friend. When I did this, I liked to put the name of the professor they know in the title of the email, so they know that this email is worth their attention. Make it clear you want to go to their school and be friendly, and try to set up a call with them if possible. All of this can add up very quickly, and lead to even bigger things than the response I got from figure 4.

5. Application is complete! What now?

Congratulations! You finished and you should be proud of yourself. The work does not necessarily end here, but you finished the biggest chunk of it. You technically do not need to do anything at this point, but the advice I give in this chapter benefited me and I recommend you follow suit. First, a timeline of what to expect.

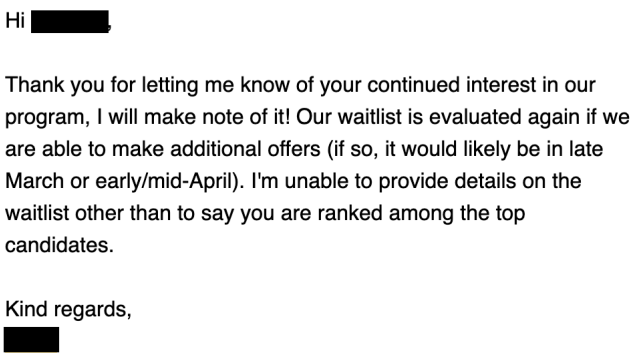
5.1. **General timeline.** The admissions process takes a while, and schools vary widely on when they give out decisions. It starts as early as January, and the admissions will roll out until mid April. **April 15th is *the* day all schools seem to agree as the final day to give out offers.** But some schools do not adhere to this and can stretch it out a bit if they deem it necessary.

Schools admit applicants in waves, and will send out admissions to their best applicants first. Similarly, they will reject the applicants they never plan on admitting first. Most likely you will receive only rejections when decisions first roll out, so don't let it discourage you. Rejections are a part of the process, and no one gets into *every* school they apply to.

Some schools never even tell you that you've been rejected, and leave you totally in the dark. This is rare but happens. This means that if you are approaching March and April and still haven't heard back from a school yet, you are most likely on their waitlist, which is great! In my experience people who are enrolled in a grad program likely got in through a waitlist, since people who got in the first wave will likely choose another "better" school.

This leaves the unfortunate scenario of people holding onto their acceptances until they get off the waitlist of somewhere "better", creating a domino effect of a lot of acceptances really close to the April 15th deadline. Even if the school's website says they give out admissions mid-March, expect the possibility of not hearing back from them until April 15th.

If it is March-April and you do not hear back from a school, it can be difficult to tell where you stand on their list. The best way (and only way I can think of) to know this is to reach out.


Hi [redacted],

Thank you for letting me know of your continued interest in our program, I will make note of it! Our waitlist is evaluated again if we are able to make additional offers (if so, it would likely be in late March or early/mid-April). I'm unable to provide details on the waitlist other than to say you are ranked among the top candidates.

Kind regards,
[redacted]

FIGURE 5. Average response when asking about your application status.

5.2. **Keep outreaching.** Yes, more emails! At this point emails will be your friend. Continue reaching out to professors. Do not be afraid to reach out to whoever is charge of admissions and directly ask them when you expect to hear back. Take this opportunity to let them know that they are high on your list and would be thrilled to receive an acceptance. And for your top school, *tell* them that they are your top school, and give reasons for why you would be delighted to attend. Be genuine and excited!

Figure 5 shows a response I got when I emailed a faculty member in graduate admissions. I mentioned that I was very interested in their school and was curious when I should expect to hear back. As you can see they were very nice and appreciated the fact that I let them know. It's easy to assume that they would seem annoyed and perceive you as impatient, but as long as you keep the email short and sweet, they will understand and be kind.

This is also the time to continue emailing professors and maintain your connection with them. Show your continued interest and say how you would be very happy to work with them. Again, don't be annoying but definitely be persistent.

5.3. **Interviews.** Some, not all, programs may ask you for an interview. If you receive one, congratulations! In my experience most people get accepted after an interview, and you will most likely be waitlisted at the very worst.

Most likely this will be a virtual interview, but I have heard of programs that fly you out to do one in-person. Regardless, you want to be presentable and calm during the interview, and confident in what you wrote in your application. Here are some questions they may ask that you should be prepared to answer:

(1) *What research are you interested in? Why?*
(2) *Could you explain your research in more detail?* (if you have done research)
(3) *What made you want to pursue a PhD?*
(4) *Why do you think you will be successful in completing our program?*
(5) *Why did you decide to apply to this school specifically?*
(6) *What makes you unique and what qualities will you bring to our department?*
(7) *What is your favorite mathematical theorem?*

There's a good chance they will not ask you all these questions, but they are pretty popular and good to think about.

Other than that, it is just like a normal interview. Prepare for it like any other. Don't be weird. Remember to smile, and don't try to act like someone you are not, they can usually see right through this. If someone asks you a question you don't understand, it cannot hurt to ask them to restate the question or just say "I don't know" if you truly do not know the answer. They will appreciate your honesty.

This is also a great time for you to ask questions of your own to whoever is interviewing you. Think of your interview less as an interrogation and more of a conversation. Here are some good ones to think about:

(1) *How supportive is the department in getting through qualifying/preliminary exams? How difficult is it to pass?*
(2) *What is the community like amongst the grad students? Amongst the whole department?*
(3) *What is the TA load? On average how many hours a week do you need to TA? When do I start TA-ing and when do I stop?*
(4) *How many students do an advisor usually take?*
(5) *How comfortable is the stipend I will receive to live in the area?*
(6) *What do students at your department tend to do after they graduate: do they tend to stay in academia or go to industry?*
(7) *What qualities do students who succeed in your department tend to have? Do you have any advice for an incoming student like myself?*

Part of an interview is them convincing *you* to go to their school, so when asking these questions make them do a lot of the talking.

5.4. **Getting an invitation to visit.** Most schools like to invite their first wave of applicants to visit, as they are often their strongest and most promising candidates. This could entail them offering to fly you out to campus and stay for several days with all expenses paid. If you are offered this opportunity I highly recommend you take it and experience the school. I was flown out to a school that was initially low on my list, but I was impressed because it was nicer than I had anticipated.

While you are there, the schools will do all they can to "roll out the red carpet" and convince you to attend their program. They will get their sweetest professors and grad students to show you around, so don't be fooled if it seems like *everyone* in the department is amazing: this is intentional.

If you get a chance, definitely try to talk to some current grad students and ask them about their experience. Ask them what it's *really* like to be a grad student, and ask about the flaws of their school/if they are happy with where they are at. They have nothing to gain by convincing you to come to their school, and will be on your side when it comes to giving advice. They understand all of your concerns: they were recently in your exact situation.

5.5. **Making a decision.** If you are in the fortunate situation to receive multiple acceptance letters, you have a big decision to make. Maybe it is an easy one, but there's a possibility it is not so clear. This is when you consult your professors, friends, and any resource you can to make your decision.

As for my advice, here is a rough list of things you should consider, ranked (in my opinion) in order of importance:

(1) Future advisor: **Your advisor will be the single most important person during your time in grad school.** If you know a professor you get along with well already, this takes priority over *everything* else.

(2) Number of professors in your research area
(3) Culture/vibe of the school
(4) Location
(5) Stipend amount (relative to cost of living in your area)
(6) Prestige/program ranking

Notice how "other people's feelings" is not on this list. **Do not consider how others feel about where you are going and make the best decision for yourself.** I got into the school that my family wanted me to go to the most, but I declined that offer in favor for the program *I* wanted to go to the most. I do not regret it one bit. Listen to what others have to say, but don't forget that you do not have to follow their advice. At the end of the day this is your decision, so go with your gut and trust yourself.

6. What to do if things don't go according to plan

It happens. This is not a reflection of you or your intelligence, but is a result of the way decisions are made. Maybe the person who looked at your application was having a bad day. Maybe you were *one* person away from getting off the waitlist. There are so many "maybes" that are out of your control. If you're in this situation, here are the two options I can think of (besides just waiting until the next application cycle).

6.1. **Master's.** Sometimes programs will offer you a master's at their university if they do not have the funding for you as a PhD student. If you have the financial means, then this is absolutely a viable option.

6.2. **5th year.** Assuming you are able to, you can purposely take a 5th year and reapply the following year. What's good is that you can still use your financial aid (assuming you still have it) to take another year, and fill it with more research and grad classes. You will have more time to retake the GRE, refine your personal statements, and will be more prepared than ever. Do an REU over the summer. Talk to more professors. I recommend leaving a GE class until your very last semester/quarter so you can "hold it hostage" and not take it in case you need to take the 5th year. Check in with a counselor before doing this as you may need to petition to be able to enroll in a 5th year.